

EXPLORATORY STUDIES OF OPEN-CHANNEL FLOW OVER BOUNDARIES OF Laterally Varying Roughness

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Pasadena, California

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by

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ABSTRACT

An exploratory study was made of open-channel flow over beds consisting either entirely or partially of large granular roughness. Steady, uniform flow was established at various depths and velocities over two types of beds, one rough over the entire width of a laboratory flume, the other rough only over half the width and smooth over the other half. Friction factors were determined for these flows, and detailed velocity distributions were measured in three runs.

The friction factors for the entirely rough beds compared closely with those predicted by the Karman-Prandtl equations, and the velocity distributions strongly suggested the existence of secondary circulation of the second kind.

Analysis is offered to show that subdivision of the cross section of a turbulent flow by curves normal to the equal velocity curves does not result in hydraulically independent zones of flow, in that there will be turbulent interchange of the longitudinal component of momentum among such zones; other methods of subdivision are considered and none found to be completely satisfactory.

The customary side-wall correction method is reviewed and found to have no explicit rational basis, and although it is recognized that the method gives reliable results in the situations to which it is usually applied, its application to widely different situations should be undertaken with caution.

Suggestions for needed further research are offered.

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I. INTRODUCTION

1. General observations

A river is a flow of water and sediment, whose behavior is governed by the water and sediment loads imposed on it and by the nature of the valley in which it flows. The regimes of rivers are continually being altered by man, by the construction or operation of dams, diversion works, or straightening and deepening operations, and the effects of such activity often extend many miles up and down the stream from the actual site of the activity. Hence, a complete assessment of the impact of such man-made changes is important to orderly planning of water resources development and control. However, mechanics of flow in rivers is still not very well understood.

The turbulent flow of water in flumes, lined canals, and other rigid conduits of homogeneous boundary roughness is well understood in its gross aspects, conceptual models of the structure of the turbulence have been fairly successful. Von Karman's similarity hypothesis, originally proposed to apply to the region of flow in the general neighborhood of the boundary, is the basis of workable and successful descriptions of the velocity distribution and over-all resistance to flow found in such channels.

Once consideration is given to channels consisting of loose, movable material (e.g., most natural streams, flowing over sediment), the picture is much less encouraging. For not only must one then investigate the rate of movement of the channel material (i.e., sediment), but one also soon realizes that the resistance-to-flow relation is much more complicated. Where in the former case a friction factor could be related to the stream Reynolds number and to an index of the roughness of the fixed boundary, now the boundary is no longer fixed, and its shape

depends on flow conditions and on the rate of sediment transport. Thus in studying alluvial streams, one seeks relations for both sediment and fluid transport (commonly referred to as "transport" and "resistance-to-flow" or "roughness" relations, respectively).

Of these two functional relations, the transport relation has had much more attention in the past, although there have been several recent efforts to understand the roughness relation. These efforts vary in sophistication and success, and will be only very briefly reviewed here. Each of the principal papers dealing with resistance to flow in alluvial open channels starts by postulating that the boundary shear stress may be considered as the sum of two parts. The first part is that shear which would be felt by a fixed, flat bed of the same texture as that of the actual bed, and is usually referred to as the "grain resistance". The remainder (the "form drag") is considered to arise principally from the effect of bed forms (ripples, dunes, etc.), although being a residual term, it necessarily includes all side effects.* The idea of dividing the shear seems to have been suggested by Meyer-Peter and Müller (1), who chose to express this division in terms of a corresponding division of the energy slope. Einstein (2,3) expressed the division in terms of two hydraulic radii, and he and Barbarossa (4) have published the analysis of some field observations on the relation between a shear parameter and the resistance to flow, in which the effect of the bed forms ("form" or "bar resistance") is expressed in terms embodying the partitioned hydraulic radius. Various workers at the Colorado State University (5, 6, and 7, for example) have written on the subject, depending heavily on dimensional analysis to order and extend the results of their laboratory work. Taylor and Brooks (8) and Taylor (9) have

*These include effects of channel alignment and changes in shape.

attempted to simplify the means of describing bed configurations, now largely a subjective process.

All the studies mentioned above have depended either on very generalized descriptions of the natural channel studied (as in Einstein and Barbarossa) or on very idealized laboratory models of natural channels. In the typical laboratory flume, experiments are frequently arranged so as to have as nearly a two-dimensional situation as possible. Such simplicity is seldom found in natural streams, it being quite common to find marked lateral variations of bed configuration and roughness in such streams, and to find those variations associated with variations in depth as well. Therefore, the extent to which the laboratory results apply to the behavior of natural streams is problematical.

In order to explore the influence of lateral variations in bed roughness, a series of flume experiments was undertaken, wherein the effect of a large and sudden lateral change in roughness could be studied. Large roughness elements were selected so as to demonstrate clearly any peculiarities in this flow situation. A comparison series was undertaken, to calibrate the roughness elements used; the results of this series were also intended to serve as a verification of the Karman-Prandtl equations for rough, two-dimensional channel flow, in a range of high values of relative roughness.

2. Flow over homogeneous roughness

It is appropriate to review some of the relevant expressions for resistance to flow in circular pipes and open channels. Since this creates the need to distinguish among friction factors calculated

for a variety of situations, the following special notation will be established.

For calculated friction factors, we define

Symbol	Description	Defining Equation
f_{so}	Flow in smooth circular pipes	3
f_{ro}	Flow in rough circular pipes	4
f_s	Flow in smooth two-dimensional channels	5
f_r	Flow in rough two-dimensional channels	6

Also, for describing rectangular channels of finite width, and for typographical simplicity, we define

W	Ratio of width, b , to depth, d	$W = b/d$
E	Relative roughness	$E = \epsilon / 4r,$

where ϵ is the height of the boundary roughness elements.

Other quantities will be defined as introduced; for completeness and convenience all symbols are defined in the Summary of Notation following the text (see page 60).

The Darcy Weisbach friction factor, f , is defined as

$$f = \frac{8grS}{u^2}, \quad (1)$$

where g is the acceleration of gravity, r is the hydraulic radius of the pipe or channel, S is its energy gradient, and u is the mean velocity.

In terms of the Reynolds number* written

*The factor four is usually included because the Reynolds number thus defined is more convenient for use with pipe-friction diagrams, the diameter of a circular pipe being four hydraulic radii. While this convention is now generally accepted usage in American hydraulic literature, it was not ever thus: Johnson (10), for example, in 58 pages of tables summarizing significant bed-load experiments, uses r as the characteristic length. Furthermore, Schlichting (11) defines the hydraulic radius as twice the area divided by the wetted perimeter, so that the hydraulic and geometric radii are equal for circular sections.

$$R = \frac{4ru}{\nu}, \quad (2)$$

ν being the kinematic viscosity of the fluid, the Karman-Prandtl equations for resistance to turbulent flow in a circular pipe may be written

$$1/\sqrt{f_{so}} = 2 \log (R \sqrt{f_{so}}) - 0.8 \text{ (smooth pipes, } \frac{u_* \epsilon}{\nu} < 3) \quad (3)$$

and

$$1/\sqrt{f_{ro}} = -2 \log E + 0.54, \text{ (rough pipes, } \frac{u_* \epsilon}{\nu} > 70) \quad (4)$$

where u_* , the shear velocity, is given by $u_* = \sqrt{grS}$.

The form of these equations arises from the assumption of a logarithmic velocity distribution; the constants are empirical and differ but slightly from those calculated on the assumption that the logarithmic velocity distribution holds throughout the main body of the flow. The limiting values of $u_* \epsilon / \nu$ are also empirical.

In the case of two-dimensional open-channel flow, there is no dependable experimental determination of the constants, so the corresponding equations are based on the logarithmic velocity distribution assumption without correction, and are customarily taken to be

$$1/\sqrt{f_s} = 2.03 \log (R \sqrt{f_s}) - 0.47 \text{ (smooth channels, } \frac{u_* \epsilon}{\nu} < 3) \quad (5)$$

and

$$1/\sqrt{f_r} = -2.03 \log E + 0.91. \text{ (rough channels, } \frac{u_* \epsilon}{\nu} > 70) \quad (6)$$

Extension of the Karman-Prandtl equations to sections which are neither circular nor two-dimensional rests on the work of Keulegan (12), who concluded that for a first approximation, channels with equal hydraulic radii may be considered equivalent as far as flow relationships are concerned. He found that for more careful work, particularly in the case of polygonal cross sections departing widely from the two-

dimensional, it was necessary to introduce a shape factor, β , into the equations corresponding to 5 and 6 above. The effect of this shape factor on the present work can be seen by using run 36 as an example (see Table 2). In this run, the width/depth ratio and relative roughness were $W = 2.32$ and $E = 0.103$. For this value of W , inclusion of β would increase the constant 0.91 in equation 6 to 1.07; for the observed value of E this change corresponds to a reduction in f from 0.122 to 0.109. For shallower runs (i.e., for larger values of W) the corresponding reduction in f would be less.

Powell (13,14) has reported some experiments in rectangular open channels, on the basis of which he takes issue with Keulegan's analysis and conclusions. In the absence of anything definitive on the question, and in view of the exploratory nature of the work reported herein, no attempt was made to include a shape-factor correction in the subsequent analysis.

In the so-called "transition zone," i.e., for $u_* \epsilon / \nu$ between about 3 and 70, the flow resistance is dependent on both the Reynolds number and the relative roughness, and in particular on the form and distribution of roughness elements. Several empirical curves exist for predicting the friction factor in the transition zone, the best known of which are those of Colebrook (15) and of Nikuradse (16). Colebrook's transition is based on experiments with commercial pipes, in which the major resistance to flow is caused by irregularities, all of which tend to be widely spaced with respect to their size; hence this transition curve may be spoken of as applying to isolated roughness elements.

Nikuradse's results were based on experiments with pipes completely lined with sand grains, and his curve may be spoken of as applying

to close-packed granular roughness elements. Colebrook and White (17) experimented with various combinations of isolated and close-packed granular roughness elements and produced transition curves ranging consistently in character between those of Nikuradse and of Colebrook.

Since the roughness elements in the present work were essentially of the close-packed granular type, the Nikuradse transition curve has been assumed. No opportunity arose to verify this assumption, however, since in all runs the gravel-covered part of the bed acted fully rough in the hydrodynamic sense.

3. Flow over boundaries of laterally varying roughness

In Section 1 above, it was stated that most laboratory experiments on the resistance to flow in open channels have been conducted under carefully two-dimensional conditions, but that these experimental conditions fail in important ways to reflect characteristics of natural streams, in which lateral variation in depth and roughness may be very pronounced.

Two examples of this lateral variation in small streams are shown in Figure 1. The data for the smaller stream, Virgin River near St. George, Utah, were included in a memorandum report (18), and those for the other, Galisteo Creek near Domingo, New Mexico, were furnished by the Albuquerque office, U. S. Geological Survey.

In both streams there is marked lateral variation in depth, and in the Virgin the deeper parts of the cross section were associated with a soft, dune-covered bed. The tendency for dune-covered beds to be softer than flat ones has been noticed before, e.g., by Simons (19). Thus, from comments regarding the softness of the sand bed in the deep parts of Galisteo Creek it would seem likely that there were dunes in the band from about 75 to 85 feet (transverse distance). Although the

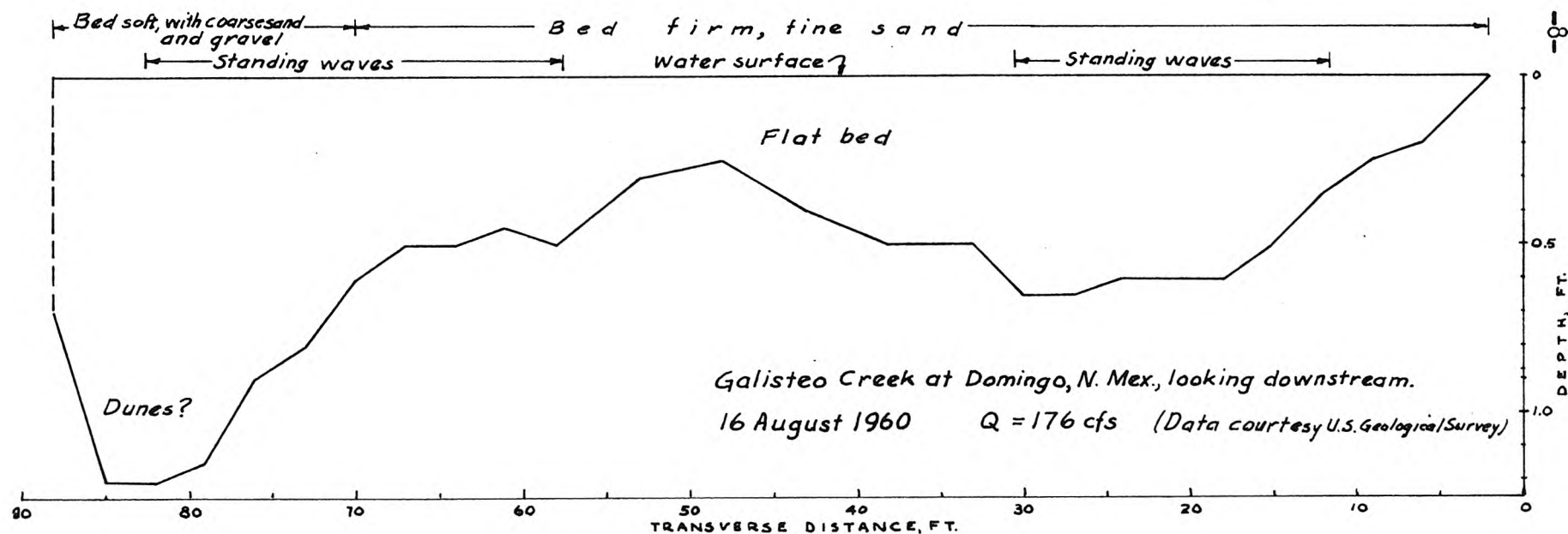
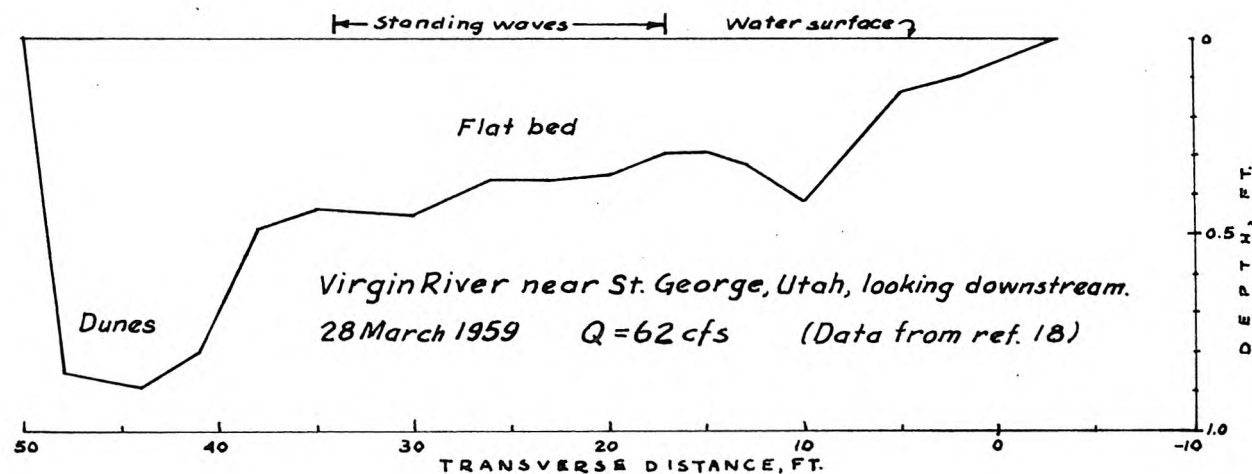


Figure 1. Cross sections of two streams which display lateral variations in depth and bed configuration.

band of standing waves extending to the left of station 70 seems inconsistent at first glance, Kennedy (20) has observed standing waves over a dune-like bed. At any rate, it is clear that in both streams there is marked lateral variation in flow conditions.

An example of similar variation in a large river is shown in Figures 2 and 3. Figure 2 shows a cross section of the Missouri River at Omaha, Nebraska, together with a general map of the river-bed topography in the vicinity of the section. From the map and section it is evident that the left side of the stream is significantly shallower and smoother than the right half. Figure 3 is a more detailed topographic map of the right side of the river just above the cross section. This map shows bed features with amplitudes ranging from three to five feet, and while their form is irregular, the contours suggest a succession of dunes or bars.* The maps are based on carefully controlled soundings by the Missouri River District, U. S. Army Corps of Engineers (21), whose permission to use the material is appreciated.

It will be noted that the relative difference in depth between the two types of bed configuration is not much less than in the much smaller streams shown in Fig. 1. It can be inferred from other reports that lateral variation in both depth and roughness are fairly common. Exner (22), for example, reported observations on a sequence of large sand bars, evenly spaced and on alternate sides of the channel, which were

*Considering the difference in mapping techniques and in the ratio of contour interval to dune amplitude, the similarity between Fig. 3 and Fig. 7 of Thompson (23) is quite striking. Thompson shows that precision photogrammetric techniques can be used in mapping bed configurations produced in laboratory flumes, and his Figure 7 is an example of a bed whose features have an amplitude of about 0.07 feet, shown in a map whose contour interval is 0.01 ft.

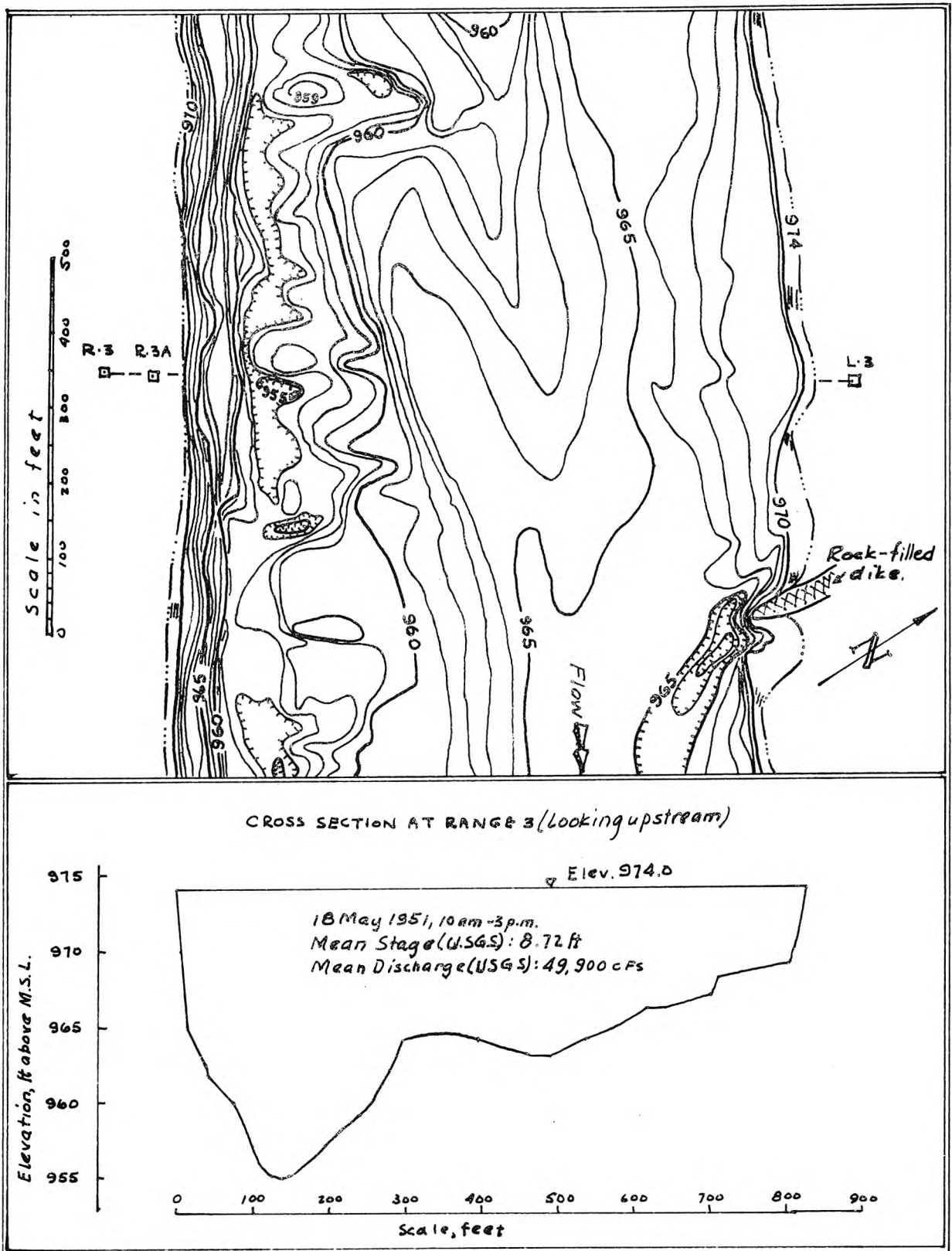


Figure 2. Bed Topography, Missouri River near Omaha, Nebraska, 18 May 1951. (Section R-3—L-3 lies approximately at river mile 640.75). After U. S. Army, Corps of Engineers (21).

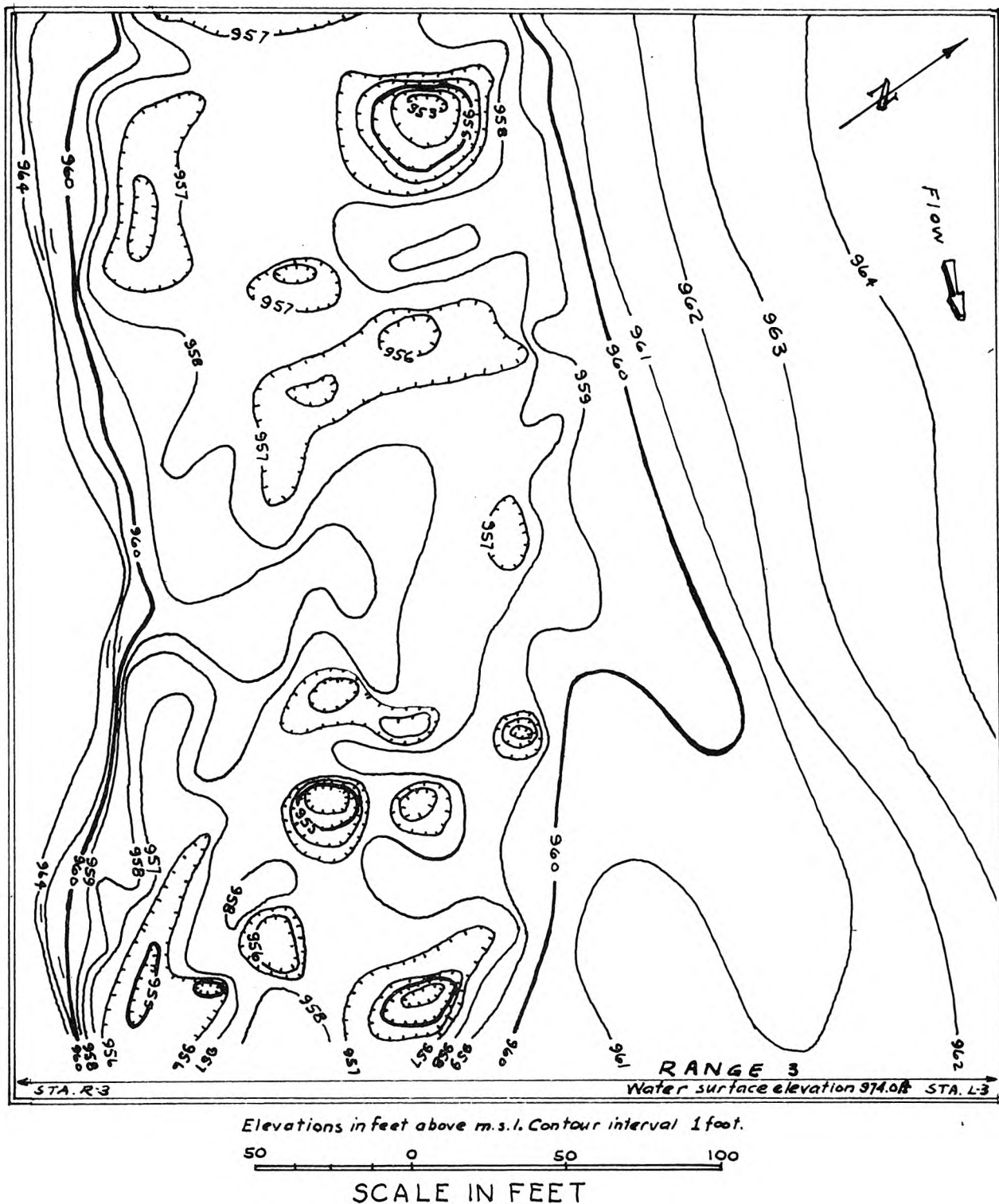


Figure 3. Detailed Bed Topography, Missouri River near Omaha, Nebraska, 17 May 1951. (Area is near right bank and just upstream from range R-3—L-3; cf. figure 2.) After U. S. Army, Corps of Engineers (21).

found in a several-kilometer length of the River Mur. His work and some others of interest are reported in Leliavsky (24).

In trying to understand such flow situations and to estimate the resistance to flow which is encountered, the question naturally arises as to the extent to which existing flume data are applicable. Can the cross section of a stream of this type be divided into segments of constant depth and bed configuration, the known results for laboratory models of such segments be applied, and a useful composite result obtained? How should this composite be obtained, and how should it reflect the interactions between adjacent segments of the original flow? The answers to these questions are not clear, but it is quite clear that if workable combining rules can be found, the usefulness of laboratory results for two-dimensional flow might be greatly extended, and the present gap between laboratory and field data reduced materially. Also, as a special case the side-wall correction procedure widely used in the reduction of flume data (see, for example, Johnson (25); cf. note, p.51) could be considered and its validity estimated.

Experiments were undertaken to examine these questions and to estimate their significance. The laboratory work, exploratory in nature, was conducted in a tilting flume 10.5 inches wide, with the gravel covering either the entire bed, or the right or left half. Since variations in depth would have complicated the problem unduly at the present stage, wood inserts were used for the smooth half of the bed to raise it to the same mean level as the gravel bed. The results of those experiments form the basis for this study.

II. EXPERIMENTAL PROCEDURE AND RESULTS

1. Statement of problem

In the interests of simplicity only two bed arrangements were considered. In the first, referred to as the "split" bed, the channel bed was hydrodynamically rough on one side of the centerline and hydrodynamically smooth on the other. In the other, referred to as the "full" bed, the bed was hydrodynamically rough over its entire width. The cross section was rectangular, and the walls were hydrodynamically smooth. All the variables considered significant for the purpose of this investigation are listed below, although for a more careful study others might be added; Figure 4 is a typical cross section and a dimension sketch.

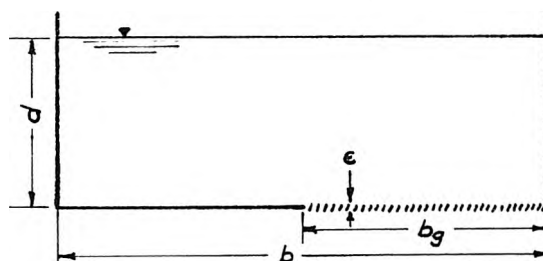


Figure 4

Variables describing channel geometry or fluid properties:

Symbol	Description	Dimensions
b	width of channel	L
b_g	width of rough part of channel	L
e	some measure of roughness height (mean grain diameter, say)	L
ν	kinematic viscosity of fluid	L^2/T
ρ	density of fluid	FT^2/L^4
g	acceleration of gravity	L/T^2

Variables pertaining principally to the flow:

Q	fluid discharge	L^3/T
u	mean velocity of flow	L/T
d	depth of flow	L
S	energy slope	-
f	Darcy-Weisbach friction factor	-

Of these eleven variables, the first six may be considered as specified for any stream or group of laboratory runs. Of the remainder, only two may be considered as independent variables, and the other three must therefore be considered dependent. For example, in the typical tilting flume experiment Q and d would be known, with u, S, and f to be determined. In a field problem, S and Q might be known, three quantities again remaining to be determined. Considering eleven variables, of which eight are known, three equations are needed. Of these three, two are known, namely the continuity equation,

$$Q = bud, \quad (7)$$

and the Darcy-Weisbach equation,

$$S = \frac{fu^2}{4r \cdot 2g} \cdot \quad (8)$$

The latter may be considered a definition of f, in which r, the hydraulic radius, is simply an abbreviation for $bd/(b + 2d)$. The third equation is an as yet unknown flow resistance, or roughness, relation.

Since Q and S are given by eq. 7 and 8, only the other nine variables need be considered in formulating the roughness relation. They may be combined into the following six convenient, independent, dimensionless groups:

$F = u/\sqrt{gd}$	Froude No.	} (9)
$R = 4ru/\nu$	Reynolds No.	
$\theta = b_g/(b + 2d)$	Roughness distribution ratio	

f	Friction Factor	} (9)
$E = \epsilon / 4r$	Relative Roughness	
$W = b/d$	Width-Depth Ratio	

Of these F is unimportant so long as there are no wave effects.

Thus, for the present study the problem consists of finding the form of the function φ in

$$f = \varphi(R, \theta, E, W). \quad (10)$$

In the simple case of flow in a pipe the parameters θ and W do not appear in the roughness relation.

It should be kept in mind that the foregoing analysis presumes no movement of the bed material. If there is sediment movement, an additional variable must be included to reflect the rate of sediment transport. There will also be variables describing the sediment itself (e.g., its density), which may be considered specified, but the rate of transport will be a variable whose inclusion will make an additional functional relation necessary for the complete solution of the problem. Furthermore, the flow resistance and the transport rate may be interdependent to the extent that it would be useless to consider one without the other. For example, the appearance of ripples or dunes on a previously flat sand bed cannot occur without some transport; however, the appearance of such features significantly alters both the flow resistance and the transport rate, which in turn influence the further development of the bed forms.

2. Objectives

The objectives of the experiments were first to determine how f varies with θ , the roughness distribution ratio, and second to study the

flow structure as revealed by detailed velocity profiles.

The principal observations made were (1) those necessary to the determination of overall friction factors for runs of both bed types over a range of depths and discharges; and (2) detailed velocity traverses necessary to determine the distribution of flow in the channel, the existence of secondary currents, and the extent to which the assumptions of the standard side-wall correction procedure were realistic in this deliberately extreme situation. The results of these observations are presented in Section 5 and some implications are discussed in subsequent chapters.

3. Laboratory procedure

The laboratory work was done in the Institute's 10.5-inch, tilting, recirculating flume (see Figure 5). This flume is 40 ft long, and has been rather completely described elsewhere, as have the standard experimental techniques used in conjunction with it; see, for example, Vanoni and Brooks (26), Brooks (27), or Kennedy (20). Suffice it to say here that the flume and all appurtenant piping are mounted on a truss supported at two points, one a fixed pivot and the other a pair of jacks; the flow is measured by a venturi meter (6 x 4½ in) in the 6-inch return line; and water surface and other detailed observations are made with respect to rails fixed parallel to the flume, with the flume slope made nearly equal to the energy slope for the flow.

The bed consisted of painted marine plywood for the smooth half of the bottom, and nominal one-inch filter gravel for the rough half, and is illustrated in Figure 6. This gravel was the 3/4- to 1¼-inch fraction of pit-run gravel from a pit located in the alluvial fan of the San Gabriel River, and the stones were nearly all sub-rounded to rounded,

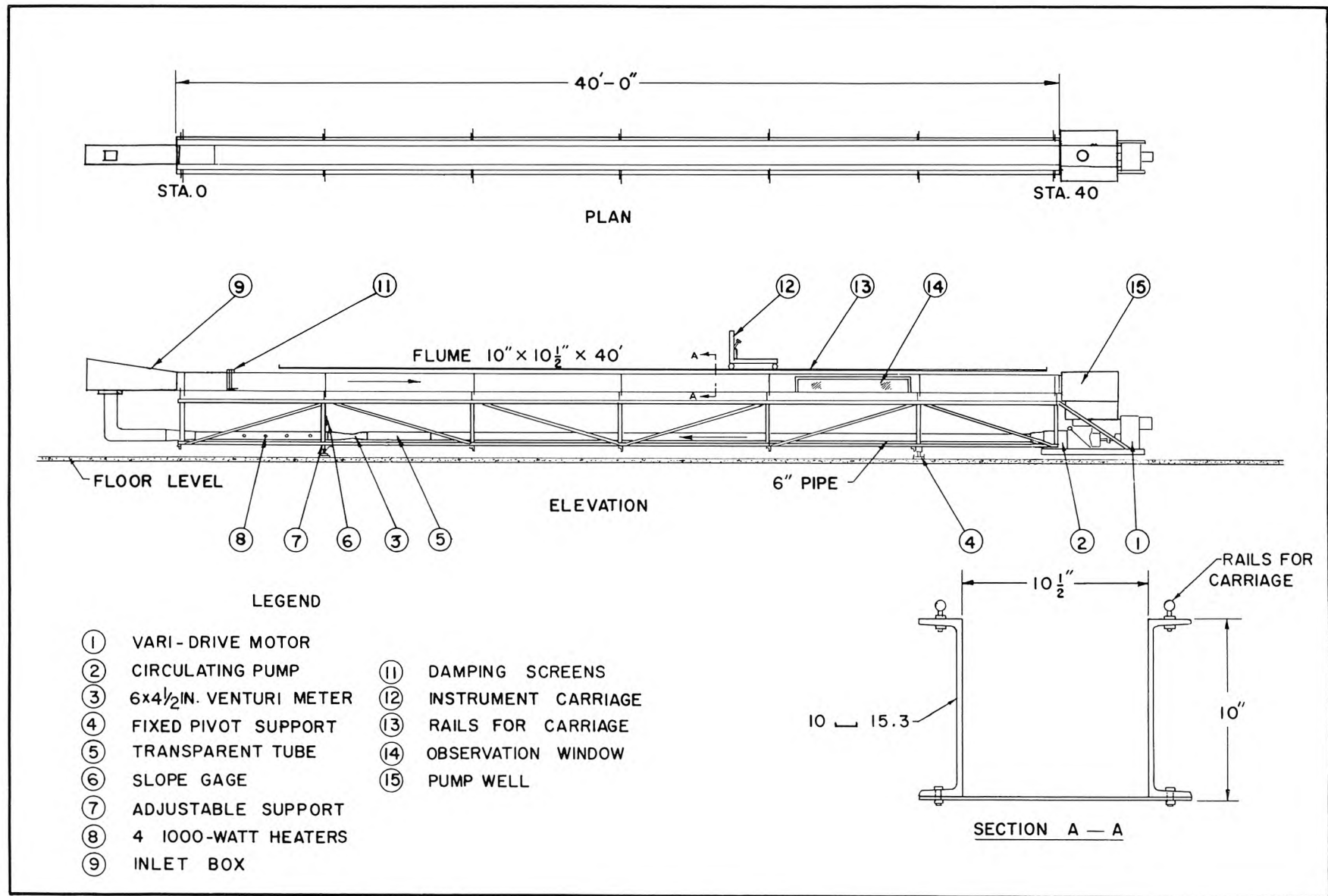


Figure 5. Schematic diagram of the flume

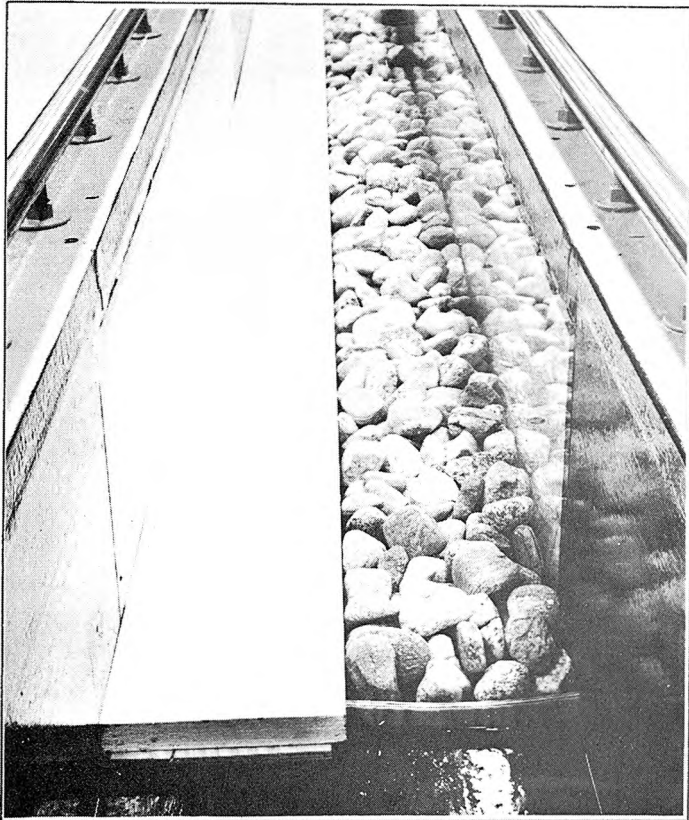


Figure 6. A typical "split" bed. The bed has been omitted in the foreground to show details of shimming and bracing.

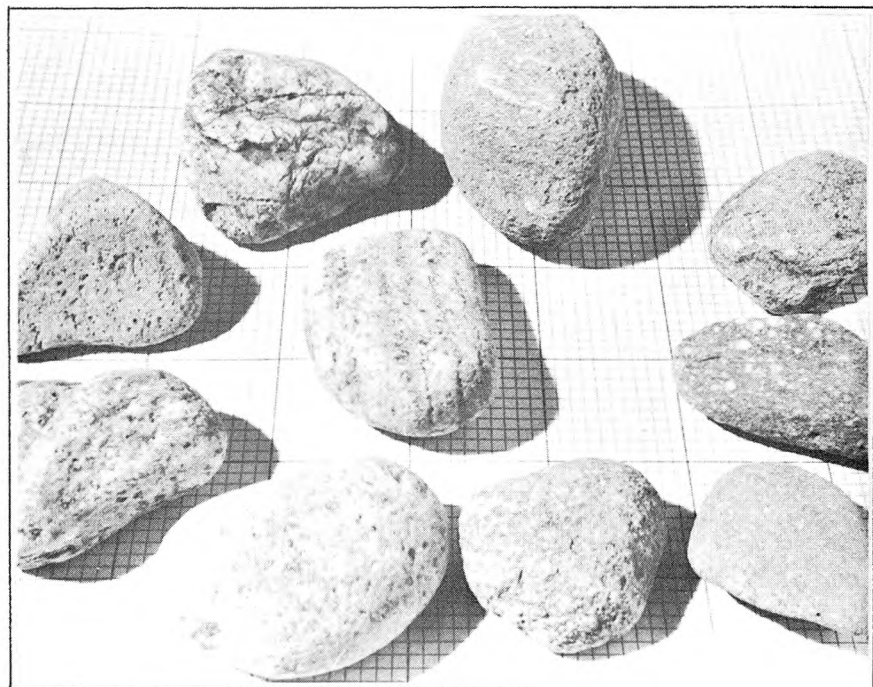


Figure 7. Sample of the gravel used. The background grid is divided in tenths of inches.

although they tended to be flattened to the extent that the ratios between principal diameters were estimated to be 1:2:4 in many cases. A random specimen of this gravel is illustrated in Figure 7.

These roughness elements were individually hand-placed at first (i.e., for runs 1 - 19.2), in order that a single layer should present a visually uniform character, but the subsequent experiments showed that hand placing did not alter the results enough to justify the effort. (This conclusion was based on consistency shown between runs 21 - 23 and 1 - 9, and was depended upon thereafter in runs 30 - 36.) Bed configurations used were 1) rough completely across the section, and 2) one half (the right side in most runs) rough, the other smooth. In the latter, or split bed case, an effort was made to make the effective depth of flow the same on both halves; however, for flow over roughness elements which may be as large as half the unobstructed depth a precise definition of depth is not straightforward.

4. Locating the bottom for the rough bed

In observing flow over a bed of fine sand, the depth of flow is ordinarily very large compared to the grain diameter, and it is unnecessary to consider flow around and between the topmost layer of grains. Furthermore, a visual averaging of the surface gives a sufficiently precise definition of "the surface of the bed". In the present work, however, one grain diameter corresponds to a substantial fraction of the total depth, and "the surface of the bed" is not a sharply defined location. Hence it must be defined arbitrarily.*

*The size of the roughness elements relative to that of the entire cross section may be seen in Figures 6, 12, and 13.

One approach would be to assume a semi-logarithmic velocity distribution and select that location which makes the straightest plot of velocity vs. \log (wall distance). This is neither precise nor productive of consistent results, because the grains are large enough that the effects of individual lee-side eddies distort the profile locally. For a bed of close-packed hemispheres, Einstein and El Samni (28) found that the logarithmic velocity distribution law is followed if distances are measured from a hypothetical wall 0.2 grain diameter down from the tops of the hemispheres, or at $y/r_o = 0.6$. The figure should be different for other shapes and arrays of roughness elements, although they found further, in experimenting on gravel, that the same result could be used provided the 65%-finer grain size is used as the effective diameter of the material.

In applying their results to hemispheres, it may be noted that the hypothetical wall is very nearly that plane at which the volume of those portions of hemispheres above the plane equals the volume of interstices between it and the equatorial plane. The plane that equates these volumes lies at $y/r_o = \pi/3\sqrt{3} = 0.604$. This is not immediately useful for gravel, however, because a uniquely defined equatorial plane does not exist. More useful is the fact that this location of the hypothetical plane is about the same as that of a plane half of whose area lies within the spheres. This plane lies at $y/r_o = \sqrt{1 - \sqrt{3}/\pi} = 0.670$.

This latter criterion, approximate as it is, lends itself to use with a bed of non-spherical grains, and produced about the same results as did the method of straightening out the velocity profile. Furthermore, since it does not involve the measurement of velocity profiles, it can be carried out more easily and accurately, and a rough visual

check is available. The procedure is as follows. Known volumes of water are added to the carefully levelled flume to cover the depth range from zero to complete submergence of the stones, and after each addition, elevations are observed at several points along the flume. Plotting the averages of these elevations against the total water added results in a stage-capacity curve. Then the desired elevation can be read from the curve, remembering that $dQ/dy = A(y)$ where $A(y)$ is the area of surface which is actually water. This process was followed at least once for each bed and before the velocity distribution measurements made in runs 22 and 23. The bed elevation for run 36 was found by locating the level required to straighten out the velocity curves on a semilogarithmic graph.

5. Summary of results

Twenty-nine friction factor determinations were made, and in three of these determinations, detailed velocity surveys were also made. The results of the friction factor determinations are presented in Table 1 and the results of the velocity surveys are given in Table 2. In the first group (run numbers below 10) the bed consisted of a single layer of gravel laid over one half and painted 3/4-inch plywood laid over the other half (i.e., a "split" bed). The gravel was placed to present a visually uniform arrangement. Hand placement was at first considered necessary because of the relatively large number of flat stones, which tended to lie flat and thereby produce areas both lower and smoother than the areas occupied by smaller, more nearly spherical stones. The same procedure was followed in placing the bed for the second group of runs, numbered 11 through 19.2, except that here the bed was gravel for the full width. For the third group, runs 21 - 23, the bed was again split, but this time about twice the depth of rock was used. It was

dumped in and generally levelled, the occasional rock which protruded being moved or replaced. The boards were shimmed up to the elevation of the hypothetical bed plane (see Section 4, above). In the last group, runs 30 - 36, the gravel was again only generally levelled, and extended over the entire width. Detailed velocity surveys were made in runs 22, 23, and 36, at stations which appeared to be free of local flow asymmetries.

The coordinate system used in recording data is indicated on each velocity distribution graph. Elevations were measured from the flume bottom plate, and transverse distances were measured from an origin nearly at the center line. Positive distances were toward the right when looking downstream, and in the split beds the rock was on the right side.

Table 1
SUMMARY OF DATA: FRICTION FACTOR DETERMINATIONS

Run No.	Depth d ft	Mean Velocity u fps	Slope S x 10 ³	Temperature T °C	Friction Factor Overall f	Friction Factor Bed f _b	Hydraulic Radius r ft	Relative Roughness E = $\epsilon/4r$	Reynolds Number R = $\frac{4ru}{v}$
<u>Group I: Split bed, one stone thick</u>									
1	.444	.955	.64	19.8	.0396	-	.220	.0946	77,300
2	.440	2.01	2.35	20.6	.0321	-	.219	.0952	165,100
3	.422	3.33	6.70	21.3	.0334	-	.215	.0970	273,000
4	.331	3.16	8.28	22.0	.0402	-	.189	.1104	231,000
5	.330	2.08	3.74	22.2	.0418	-	.188	.1108	152,000
7	.221	2.70	9.33	21.9	.0484	-	.147	.1418	153,200
8	.216	1.38	2.65	22.2	.0514	-	.144	.1448	77,400
9	.218	.805	.97	22.6	.0558	-	.145	.1436	45,800
<u>Group II: Bed entirely rough, one stone thick</u>									
11	.447	2.61	6.13	21.3	.0511	.0841	.221	.0942	220,000
12	.446	2.11	3.99	22.1	.0509	.0829	.221	.0942	180,000
13	.456	.997	.96	22.8	.0553	.0896	.223	.0934	87,700
14	.308	2.94	14.59	22.6	.0785	.1195	.182	.1146	210,000
15	.313	1.96	6.24	22.4	.0762	.1152	.183	.1140	140,000
16	.320	.979	1.59	22.2	.0786	.1175	.185	.1128	70,500
17	.147	1.092	6.36	21.8	.1508	.1910	.110	.1896	46,300
18	.156	2.04	18.15	21.4	.1279	.1640	.115	.1808	89,700
19	.156	2.77	32.21	20.9	.1238	.1587	.115	.1808	120,200
19.1	.209	3.52	32.65	20.9	.0960	.1316	.141	.1480	187,700
19.2	.261	3.82	32.42	20.9	.0933	.1367	.163	.1280	235,300
<u>Group III: Split bed, several stones thick</u>									
21	.316	2.12	4.15	23.2	.0435	-	.186	.1120	156,600
22	.297	3.04	10.0	23.3	.0493	-	.178	.1172	215,300
23	.154	3.30	25.2	25.6	.0678	-	.114	.1830	157,700
<u>Group IV: Bed entirely rough, several stones thick</u>									
30	.127	2.00	19.3	23.6	.1230	.1509	.099	.210	79,400
31	.180	2.08	13.8	23.0	.1043	.1370	.127	.1640	104,600
32	.222	2.27	12.4	22.9	.0911	.1258	.147	.1418	132,000
33	.255	2.45	11.1	24.4	.0767	.1091	.161	.1294	161,300
34	.294	2.55	10.4	23.7	.0725	.1074	.176	.1183	180,500
35	.334	2.61	9.29	23.5	.0665	.1021	.189	.1102	197,400
36	.377	2.64	8.34	22.7	.0620	.0986	.202	.1031	209,700

Table 2

SUMMARY OF DATA: VELOCITY DISTRIBUTIONS
Runs 22 and 23

RUN 22, STATION 24, (Cf. Figure 11)

Transverse Station, feet

-.415 -.400 -.250 -.200 -.150 -.100 -.050 0 +.100 +.200 +.400 +.415

Elevation ft	Velocities in feet per second											
	(Left side, smooth)						(Right side, rough)					
.370	2.81	2.98	3.62	3.76	3.89	3.99	4.00	3.79	3.27	2.70	1.88	1.72
.350	2.96	3.13	3.69	3.82	3.91	4.01	4.03	3.87	3.40	2.85	1.93	1.79
.300	3.20	3.33	3.86	3.91	3.97	4.04	4.05	3.91	3.54	3.13	1.93	1.81
.250	3.21	3.39	3.82	3.90	3.93	3.96	3.91	3.79	3.47	3.10	1.78	1.52
.200	3.13	3.26	3.68	3.73	3.69	3.70	3.68	3.36	2.98	2.45	1.44	1.21
.150	2.74	2.99	3.41	3.38	3.30	3.27	3.01	2.77	2.21	-	1.08	0.84
.125	2.47	2.60	2.63	3.09	3.02	2.94	2.66	-	-	-	-	-

RUN 23, STATION 27.5 (Cf. figure 12)

Transverse Station, feet

-.400 -.200 -.100 -.050 0 +.050 +.200 +.400

Elevation ft	Velocities in feet per second							
	(Left side, smooth)				(Right side, rough)			
.230	-	5.34	-	-	-	-	-	-
.220	4.34	5.42	5.14	4.87	4.36	3.94	2.87	2.03
.200	4.37	5.31	5.19	4.90	4.30	3.72	2.76	2.01
.175	-	-	-	-	3.94	3.16	2.47	1.91
.150	4.33	4.71	4.59	4.30	3.36	2.46	2.08	1.74
.125	3.92	4.30	4.00	3.71	2.70	-	-	-

Table 2 (continued)

SUMMARY OF DATA: VELOCITY DISTRIBUTIONS

RUN 36, STATION 8

Velocities in feet per second

VERTICAL TRAVERSES

Elevation feet	Transverse station, feet				
	Left side			Right side	
	-.416	-.300	0	+.300	+.399
.493	2.34	2.71	3.31	2.84	2.43
.468	2.45	2.87	-	2.94	2.52
.443	2.50	2.97	3.29	3.01	2.54
.418	2.56	3.06	-	3.06	2.58
.393	2.55	3.15	3.16	3.11	2.56
.368	2.54	3.18	-	3.11	2.54
.343	2.52	3.21	2.92	3.10	2.49
.318	2.47	3.14	-	3.05	2.45
.293	2.39	3.10	2.59	3.01	2.41
.268	2.28	2.99	-	2.93	2.37
.243	2.18	2.89	2.27	2.78	2.21
.218	1.81	2.71	-	2.64	2.03
.193	1.21	2.47	1.87	2.35	1.64
.183	0.95	-	-	-	1.48
.173	0.76	1.90	1.56	2.01	1.32
.163	-	-	1.50	-	-
.153	-	-	1.24	-	-

HORIZONTAL TRAVERSES

Trans- verse station ft	Elevation, feet			
	.493	.343	.243	.193
Right side				
0	3.32	2.87	2.26	1.85
.10	3.21	2.88	2.30	1.97
.15	-	3.01	2.39	1.91
.20	3.16	3.09	2.58	2.26
.25	-	3.11	2.71	2.34
.30	2.87	3.11	2.79	2.35
.34	-	2.95	2.71	2.18
.38	2.61	2.69	2.47	1.91
.40	-	2.47	2.21	1.70
.41	-	2.35	2.06	1.48
.42	-	2.18	1.83	1.34
(Rt. wall at .429 ft)				
Left side				
0	3.31	2.88	2.27	1.76
-.10	3.22	2.99	2.42	1.62
-.15	-	3.10	2.51	1.81
-.20	3.02	3.15	2.60	1.76
-.25	-	3.23	2.73	1.81
-.30	2.80	3.23	2.88	2.37
-.34	-	3.11	2.88	2.50
-.38	2.58	2.90	2.63	2.26
-.40	-	2.74	2.43	1.76
-.41	2.42	2.63	2.28	1.46
-.42	2.35	2.49	2.27	1.05
-.428	2.23	2.33	1.91	0.80
-.435	2.14	-	-	-
-.436	-	2.18	-	-
-.438	-	-	1.70	-
-.440	-	-	-	0.44
(Left wall at -.446 ft)				

III. DISCUSSION OF RESULTS: FRICTION FACTOR DETERMINATIONS

1. General observations

The values of overall friction factors from Table 1 are plotted in Fig. 8 on an extrapolated Nikuradse pipe-friction diagram, in order to give a clearer idea of the range of Reynolds numbers and friction factors represented by the data. The roughness height, ϵ , was taken as 1 inch for all bed configurations. It will be noted from this figure that all data lie in or very near the range for which the flow is considered fully rough. Thus the Reynolds number effect is minor. As expected, the friction factors were lower in the split bed runs than in the rough bed runs. The observed over-all friction factors for flows over rough and split beds will be denoted f_R and f_S , respectively.

In Figure 9 the Reynolds number effect is neglected, the plot being simply one of friction factor vs. relative roughness. In addition two sets of curves are shown as generally limiting cases. The first of these is based on the Karman Prandtl equation for turbulent flow in a fully rough two-dimensional channel (equation 6), and the second set is for flow in smooth two-dimensional channels (equation 5), at Reynolds numbers of 40,000, 100,000, and 250,000, which cover the range of values observed in the present investigation. Here, the points for the split beds fall quite nicely between those limiting cases, and tend regularly away from the smooth limiting curves as the relative roughness increases. A constant value of ϵ was assumed since the same roughness elements were used for all runs; hence, increasing the relative roughness means decreasing the depth and therefore increasing θ , the fraction of the perimeter which was rough. Thus the upward drift away from the smooth curves

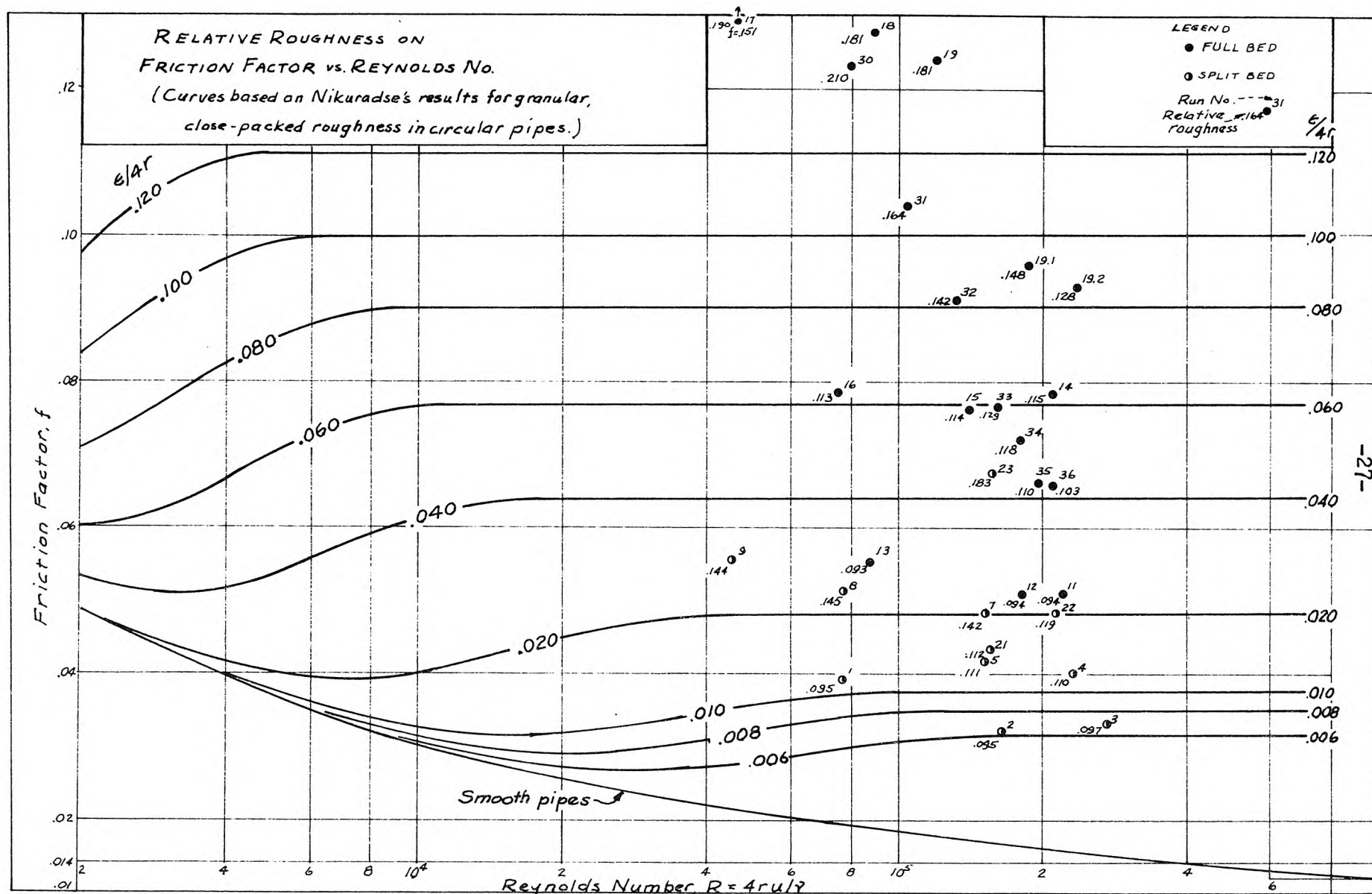


Figure 8. Experimental results plotted on (extrapolated) Nikuradse pipe friction diagram.

FRICTION FACTOR vs. RELATIVE ROUGHNESS

Full circles: Runs with entirely rough beds
 Half open circles: Runs with split beds
 Points are labeled with run numbers

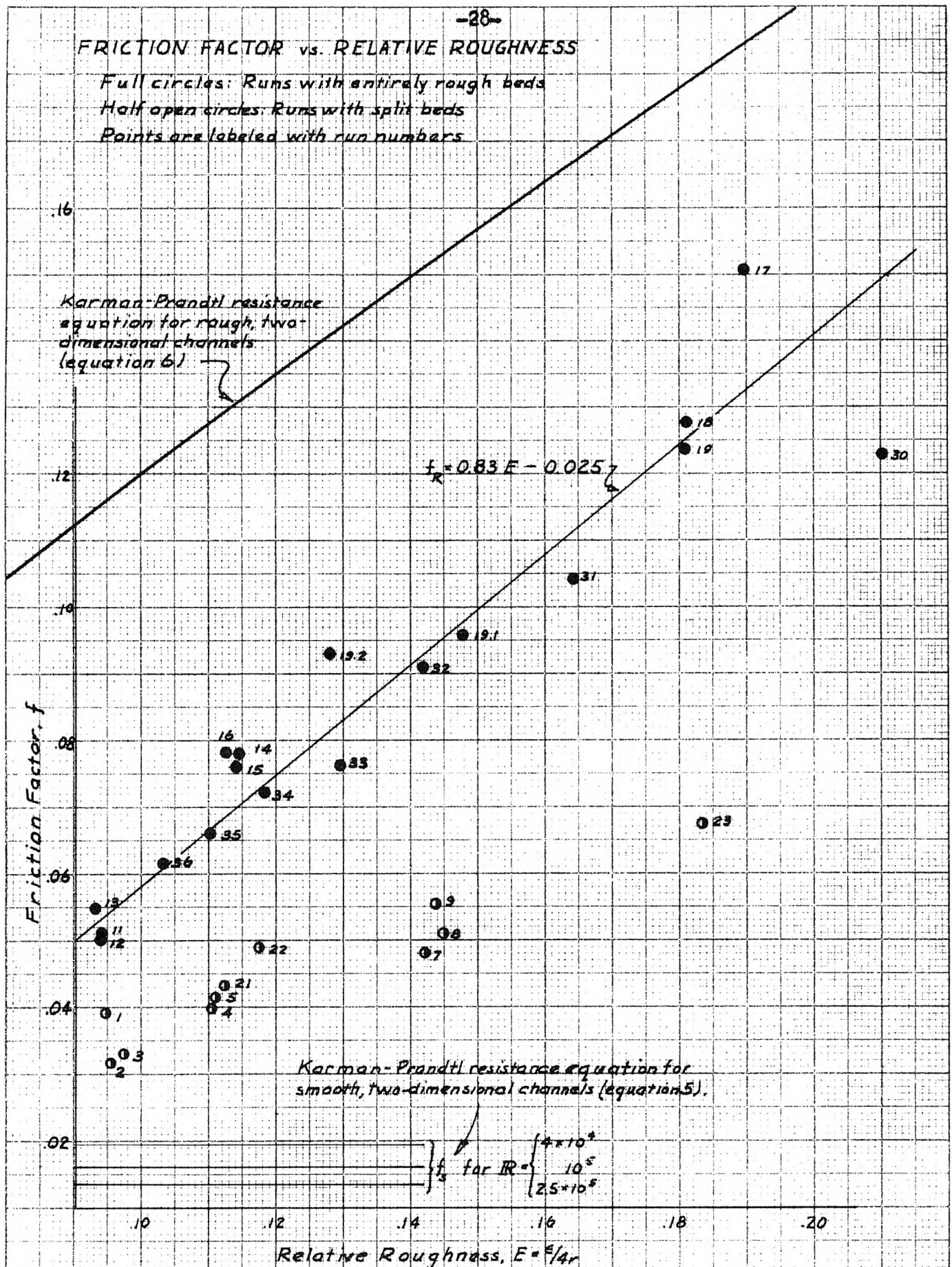


Figure 9. Friction factors vs. relative roughness.

is intuitively reasonable.

The points for the rough bed runs fall along a line approximately parallel to that of equation 6, although below it by a distance $\Delta f = 0.06$. While this difference is about constant, it represents a decreasing proportion of the observed friction factor as the relative roughness increases, or, perhaps more significantly, as the hydraulic radius (and therefore the smooth portion of the perimeter, $1 - \theta$) decreases.

If the customary side-wall correction procedure is applied to the f_R values, as outlined in Chapter V, Section 2, the agreement between them and equation 6 is very much improved, as shown in Figure 10. Observed deviations may be attributable to several things, including the extrapolation of the Karman-Prandtl equation, the doubtful assumptions in the side-wall correction procedure, the very large values of the relative roughness (Nikuradse's largest value was 0.036), and uncertainty as to the location of the "effective" bed, as discussed above. Considering the many uncertainties involved, the agreement is considered very good.

2. Comparison with existing equations

In order to examine the significance of the friction factor determinations, attempts were made to synthesize corresponding values from equations 5 and 6. It is believed that a comparison of the observed and calculated (interpolated) values offers an assessment of the internal consistency of the experimental data and may suggest procedures to be followed when a field problem requires an estimate of the friction factor in a channel of laterally varying roughness. (The question of field procedures is discussed further in Chapter V.)

Two comparison methods were tried.

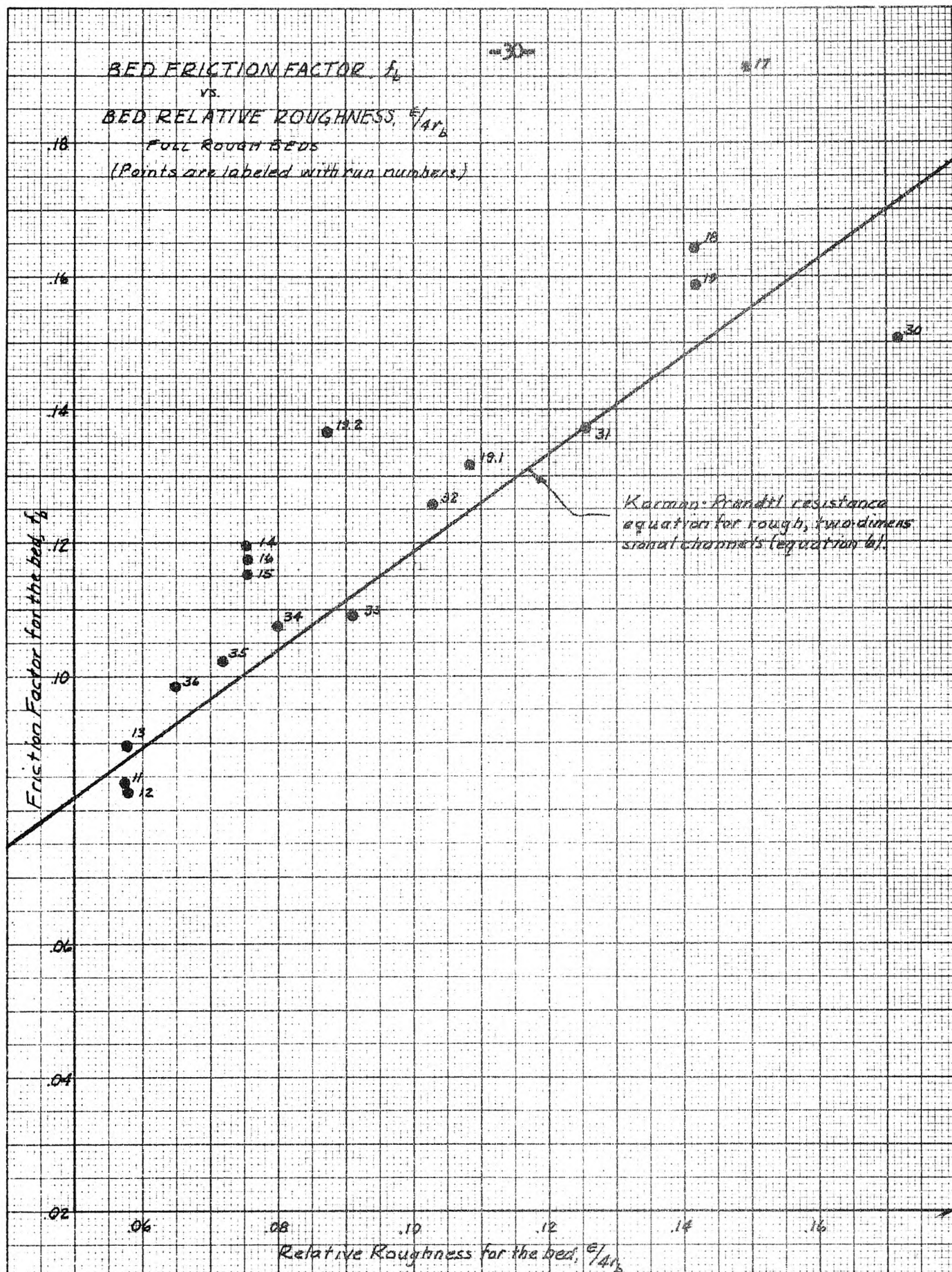


Figure 10. Friction factors vs. relative roughness (after side-wall correction).

Under Method A, it was assumed that the friction factor for the flows over the split bed should be a weighted average between (1) the friction factor for flow over a fully rough bed (for the same depth), f_R , and (2) that for flow over a completely smooth bed, f_S , as estimated by equation 5 (for the same hydraulic radius). It was further assumed that the weighting should be in proportion to the rough and smooth areas of bed. Thus the interpolation is between the friction factor for a channel with smooth walls and rough bed and one for a channel with smooth walls and smooth bed. If this interpolated value is denoted f_A , then

$$f_A = \frac{1}{2} (f_R + f_S). \quad (11)$$

The results of interpolation under Method A are given in Table 3 and are quite close to the observed values. This close agreement demonstrates the internal consistency of the data, if Keulegan's assumption is accepted. The method could be used in predicting friction factors, if f_R were estimated by calculating the f for the bed roughness at hand, and using Johnson's side-wall correction procedure in reverse, to add in side-wall effects and thus to obtain a friction factor for a rectangular channel with smooth walls and the desired bed roughness.

In Method B the attempt was made to synthesize friction factors for runs with both the full rough bed and the split bed, by combining the friction factors given by equations 5 and 6 in the same proportion in which the total perimeter was composed of rough or smooth portions. That is, since θ is the ratio of the width of rock bed to the total perimeter, the interpolated friction factor by Method B, denoted by f_B , is given by

$$f_B = \theta f_r + (1 - \theta) f_s. \quad (12)$$

The results of this scheme of interpolation are given in Table 4

Table 3. Friction factors calculated by Method A.

Friction factors					
Run No.	Observed for rough beds*	Calculated (eq. 5)	Calculated (eq. 11)	Observed	$\frac{f_A}{f_S}$
	f_R	f_s	f_A	f_S	
1	.0536	.0170	.0353	.0396	0.892
2	.0540	.0146	.0343	.0321	1.069
3	.0555	.0133	.0344	.0334	1.030
4	.0666	.0137	.0402	.0402	1.000
5	.0670	.0149	.0420	.0418	1.005
7	.0928	.0149	.0539	.0484	1.113
8	.0952	.0170	.0563	.0514	1.095
9	.0942	.0190	.0566	.0558	1.013
21	.0679	.0148	.0414	.0435	0.953
22	.0722	.0139	.0431	.0493	0.872
23	.1269	.0148	.0709	.0678	1.046
				Mean:	1.008
				Mean of deviations from unity:	.059

* Taken from line in figure 9, equation for which is $f_R = 0.83E - 0.025$

Table 4. Friction factors calculated by Method B.

Friction Factors						
Run No.	$1 - \theta$	Calculated (eq. 6) f_r	Calculated (eq. 5) f_s	Calculated (eq. 12) f_B	Observed f_R or f_S	$\frac{f_B}{f_R \text{ or } f_S}$
Split beds						
1	.754	.1157	.0170	.0413	.0396	1.043
2	.753	.1162	.0146	.0397	.0321	1.237
3	.748	.1175	.0133	.0395	.0334	1.183
4	.718	.1278	.0137	.0459	.0402	1.143
5	.717	.1281	.0149	.0470	.0418	1.124
7	.670	.1507	.0149	.0597	.0484	1.233
8	.668	.1328	.0170	.0554	.0514	1.077
9	.669	.1519	.0190	.0629	.0558	1.127
21	.689	.1290	.0148	.0504	.0435	1.159
22	.704	.1330	.0139	.0492	.0493	0.998
23	.634	.1800	.0148	.0753	.0678	1.110
Mean:						1.130
Mean of deviations from unity:						.130
Full rough beds						
11	.505	.1152	.0138	.0640	.0511	1.253
12	.504	.1152	.0144	.0644	.0509	1.266
13	.510	.1148	.0166	.0647	.0553	1.169
14	.413	.1309	.0140	.0826	.0785	1.052
15	.417	.1305	.0151	.0824	.0762	1.081
16	.422	.1297	.0174	.0822	.0786	1.047
17	.251	.1847	.0190	.1430	.1508	0.948
18	.262	.1785	.0165	.1361	.1279	1.064
19	.262	.1785	.0159	.1360	.1238	1.098
19.1	.323	.1550	.0143	.1095	.0960	1.140
19.2	.373	.1407	.0137	.0932	.0933	0.999
30	.225	.1994	.0169	.1585	.1230	1.289
31	.291	.1667	.0160	.1229	.1043	1.178
32	.336	.1507	.0153	.1051	.0911	1.154
33	.368	.1417	.0157	.0952	.0767	1.242
34	.402	.1337	.0144	.0856	.0725	1.181
35	.433	.1278	.0141	.0785	.0665	1.181
36	.462	.1223	.0140	.0723	.0620	1.167
Mean:						1.087
Mean of deviations from unity:						.145

from which it will be noted that the mean of the ratio of f_B to the observed friction factor is 1.101 for all 29 runs, being higher for the split-bed runs than for the full bed runs. Thus Method B produces results in greater disagreement with the observed ones than do the other schemes. This indicates that the effect of the smooth side walls does not go as the proportion they bear to the total wetted perimeters; comparison with the results of Method A show that this effect seems to go more nearly with the distribution of roughness on the bed only, rather than on the entire perimeter. Both methods rely on equation 5, which is relatively well established, but Method B relies also on equation 6. While this reliance on equation 6 requires using it in the range of very large values of relative roughness, this seems to be justified by the agreement noted above (Cf. Figure 10) between it and the experimental results for the full bed runs, if the side-wall correction method of Johnson is applied to the latter.

IV. DISCUSSION OF RESULTS: VELOCITY DISTRIBUTIONS

1. General observations

The velocity distributions in runs 22, 23, and 36 (Table 2) are plotted in Figures 11, 12, and 13, respectively. It is apparent in all three figures that the thread of maximum downstream velocity is located significantly below the surface. Also, in run 36 the displacement of equal-velocity curves (isotachs) toward the corners is very pronounced. There could be a similar such displacement in the other runs, but the velocity traverses are not well enough spaced to show whether this occurred or not. These features are thought to be the result of a pronounced system of secondary circulation, and will be discussed further in the next section.

In runs 22 and 23, the effect of the split bed in the velocity distribution is qualitatively what one would expect. The mean velocity of the flow over the smooth bed is quite a bit higher than that over the rough bed, and the shallower the flow is, the more nearly is the core of high velocity centered over the smooth bed. Also, the fact that the isotachs are fairly steeply sloping in the region above the break in bed roughness confirms what would be suspected intuitively, that there is considerable longitudinal shear on the vertical plane between the smooth and rough halves of the channel. It is not so obvious, however, why the maximum velocity in a given vertical section should be farther below the surface over the smooth side of the bed than it is over the rough side. As will be discussed in the next section, this is consistent with Prandtl's "secondary circulation of the second kind".

2. Evidence of secondary circulation

Prandtl (29), in discussing the velocity distributions obtained

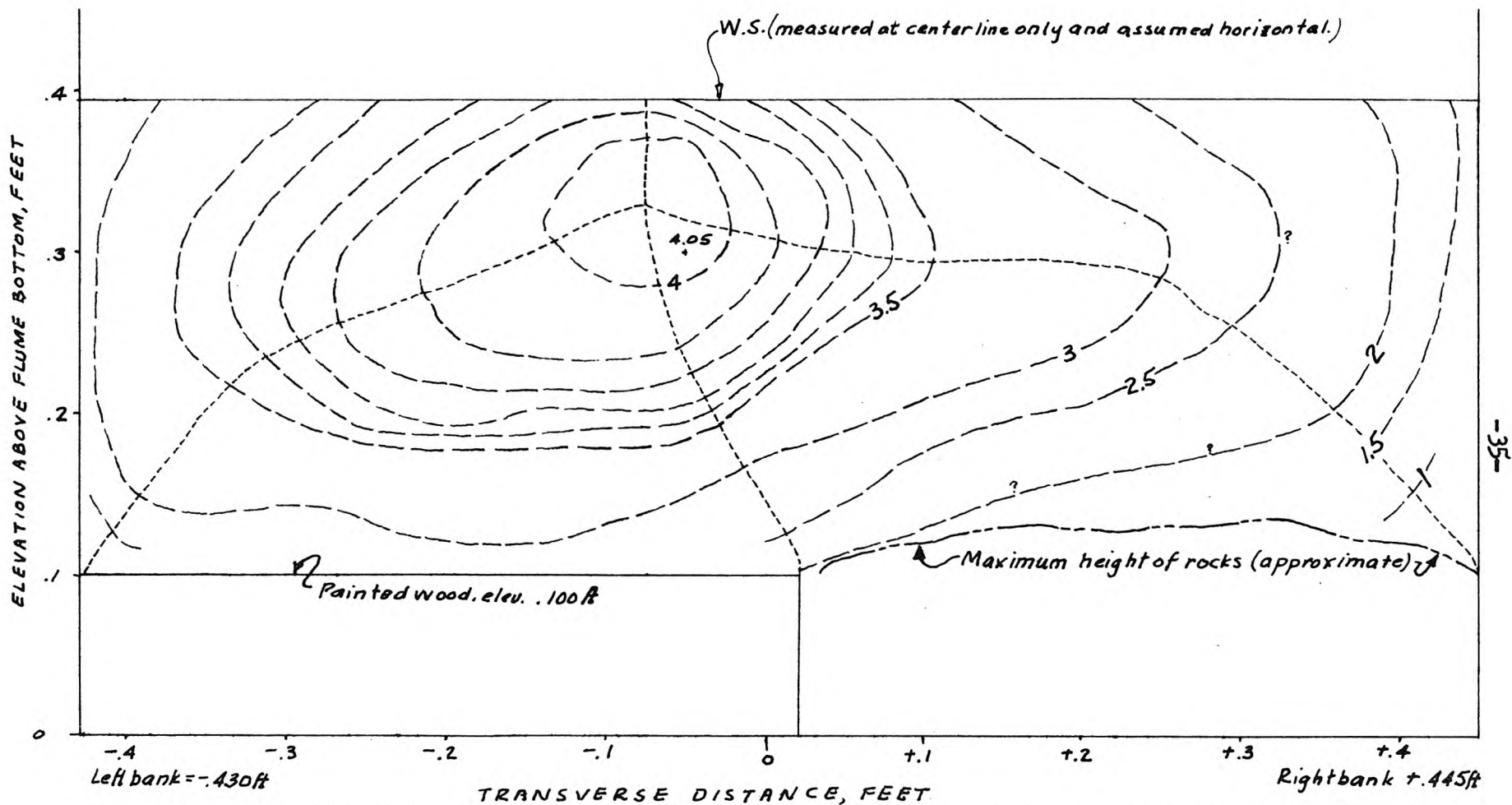


Figure 11. Velocity distribution, run 22, station 24. Velocities in feet per second; scale, 1 inch in figure equals 0.1 foot in flume. Zone boundaries for Leighly method indicated -----. Looking downstream.

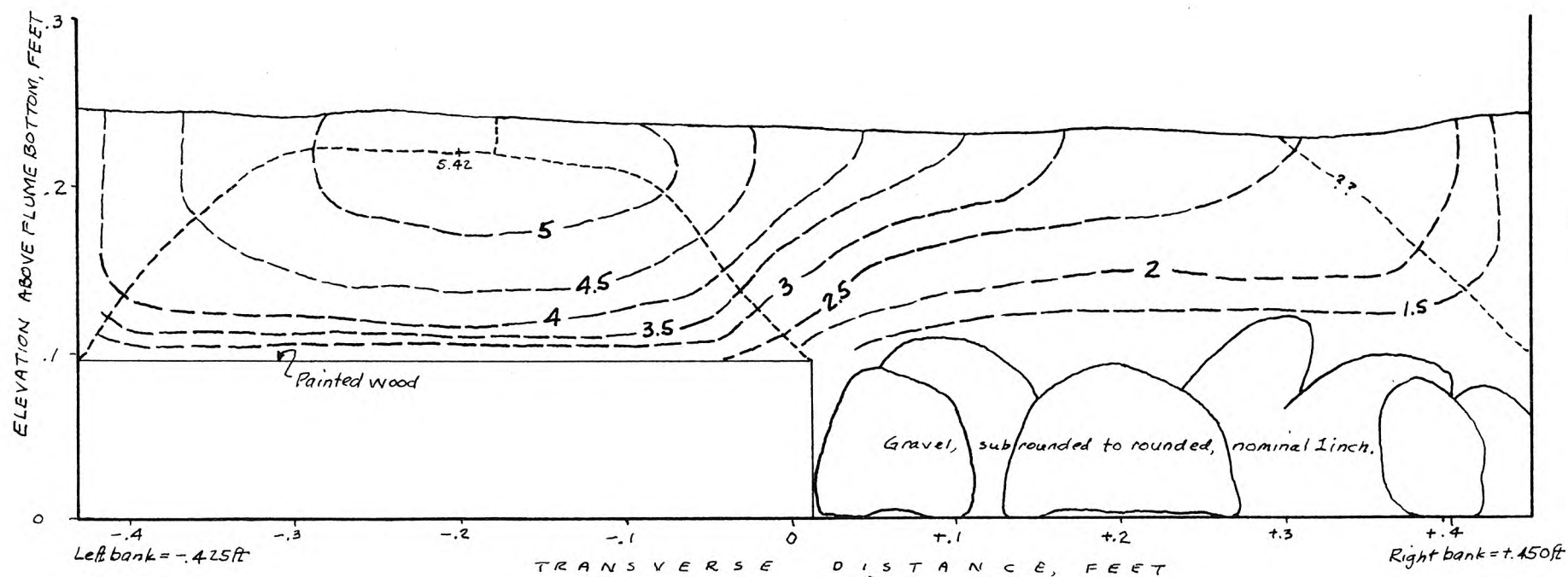


Figure 12. Velocity distribution, run 23, station 27.5. Velocities in feet per second; scale 1 inch in figure equals 0.1 foot in flume. Zone boundaries for Leighly method indicated ————. Looking downstream.

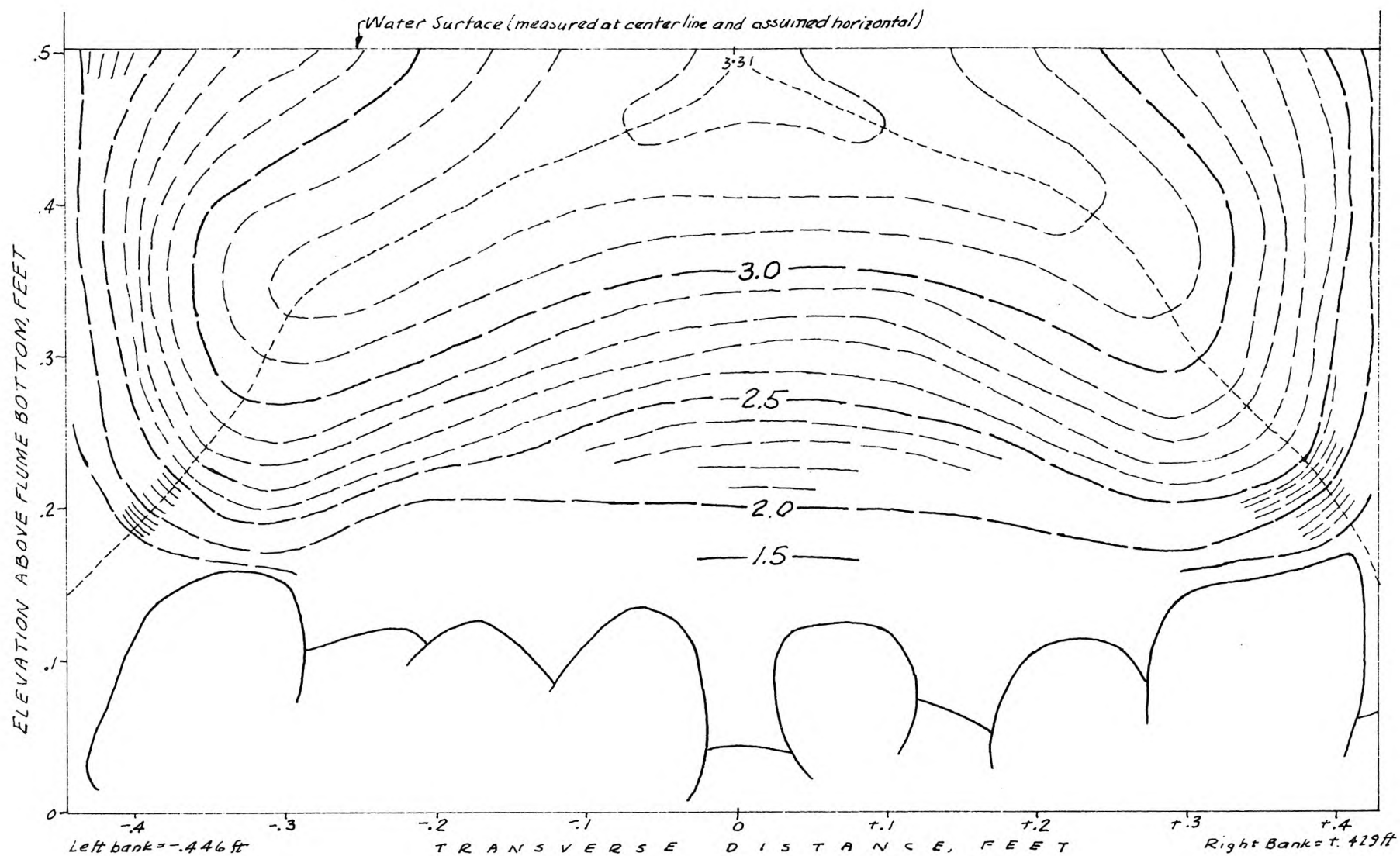


Figure 13. Velocity distribution, run 36, station 8.0. Velocities in feet per second, scale, 1 inch in figure equals 0.1 foot in flume. Zone boundaries for Leighly method indicated ----- . Looking downstream.

by Nikuradse (30) in experiments with turbulent flow in straight channels of non-circular cross section, took note of two characteristics of those distributions which are also present in the results of run 36 of the present study. They are the displacement of isotachs toward corners of the cross section, and the occurrence of the thread of maximum velocity, in open channel flows, at a point below the free surface. Prandtl's explanation is that somehow there is a pattern of secondary circulation ("of the second kind", since the more easily explained secondary flows arising from bends in the channel had already been discussed) which proceeds from zones of high velocity outward toward the corners, somewhat in the manner indicated in Figure 14. This flow, it is argued, tends to

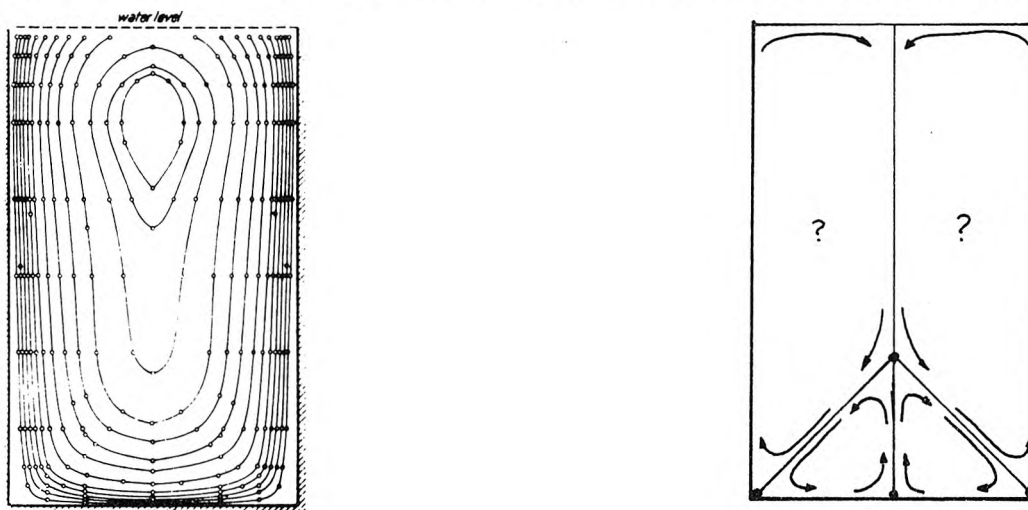


Figure 14. An example of secondary circulation of the second kind. After Prandtl (29) and Nikuradse (30).

carry fast-moving water into the areas near the corners, and to bring slow-moving water out toward the center from a neighboring section of the perimeter. The uppermost cells of this circulation would tend to bring slower-moving water in from the walls toward the center, and thus to reduce the velocity at the center of the free surface to a value slightly less than that at points just below.

Very few laboratory studies of this type of flow have been reported, Nikuradse's paper still being the principal reference. Two theoretical studies are of interest, however, those of Howarth (31) and Einstein and Li (32). Howarth deduced the condition for existence of this type of flow in pipes from the modified vorticity transfer model for turbulent flow, using Goldstein's assumed form for the mixture length tensor. He concluded that "secondary motion arises if the mixture length is not constant on the curves along which $|\text{grad } \bar{u}|$ is constant, \bar{u} being the mean velocity parallel to the pipe axis. Einstein and Li proceed from the Navier-Stokes equations as applied to uniform flow, and write an expression for the time derivative of the downstream component of vorticity. This expression is in terms of those Reynolds stresses which do not contain downstream velocity fluctuations. They argue that the vanishing of this expression is equivalent to the absence of a spontaneous occurrence of secondary flow. Acknowledging that this is a rather weak approach to the problem, they then consider conditions under which their function is non-vanishing. Among other things, they conclude that "secondary flows do not develop spontaneously in a laminar, straight, uniform flow. In turbulent flows secondary currents . . . may be expected to occur particularly near the frictional boundary where the lines of constant velocity are not parallel."

In speculating about the pattern of secondary circulation, one is led to the consideration of individual helices, or cells of circulation of a particular sense (i.e., clockwise, or counterclockwise). These cells correspond to areas of the cross section throughout which the vorticity does not change sign, and are therefore bounded by lines along which the vorticity vanishes. These lines are separation streamlines with respect to the transverse components of such helical flow

(i.e., for the yz-plane, if x is taken in the downstream direction), and contain all the "stagnation points".* Some idea could be gained of the nature of the circulation pattern if it were possible to state general properties of these lines.

It has been observed that the secondary flow moves toward corners whose walls were equally rough, and one would expect that in the immediate neighborhood of such a corner the direction of flow would be along the bisector of the corner angle. This much can be argued from considerations of symmetry about such a bisector; this same symmetry requires that the vorticity must be zero along the bisector in the neighborhood of the corner. Hence, the bisector is a separation streamline, and the corner is a stagnation point. (This is clearly indicated in reference 12, particularly in the closed triangular pipe illustrated in figure 20.15.)

The observation of Einstein and Li that their expression may not vanish "if the flow pattern changes as one follows the boundary in the cross section" suggests that there would be a circulation across any point of roughness change not at a corner. This follows because of the difference in velocity profiles to be expected over the boundary segments of differing roughness. Thus, in the split bed situation, the corner between wall and smooth bed would be a stagnation point, whereas the midpoint of the bed would not.

At present, little can be said beyond these suggestions, because

*Reference to Figure 14 may clarify this line of argument: that figure illustrates four cells, adjacent ones being of opposite sign or sense. These cells are divided by separation streamlines, and the points indicated by large dots are stagnation points, with respect to transverse and vertical (i.e., y- and z-) components of velocity.

not enough is known of the spatial variations of the mixture length (for Howarth's theory) or of the Reynolds stresses within the flows ordinarily encountered in open channels. On the basis of what has been suggested, however, a few qualitative comments may be made concerning the possible nature of the secondary circulation in runs 22 and 36. Run 23 is believed to be too shallow to allow the development of a secondary circulation with large enough or strong enough cells to make any obvious sort of modification of the isotachs.

In run 36, the isotachs are drawn much more markedly into the corners than in Nikuradse's cases (Cf. ref. 11, figs. 20.13 and 20.16), from which it seems that the strength of the secondary circulation is considerably greater in the present case. It is suggested that this greater strength can be associated with the great difference in roughness on the two walls making up the corner. This suggestion is consistent with the velocity distribution in run 22, if it is assumed that the depression of the thread of maximum velocity in a vertical is also a measure of the strength of the secondary circulation, because it appears that the level of maximum velocity does drop somewhat as one moves toward the right wall (from transverse station $-.10$ ft to about $+.25$ ft). In conclusion, only tentative suggestions can be offered as to the pattern of secondary circulation in the observed runs, but it seems very likely that a secondary circulation "of the second kind" did in fact exist.

3. Estimation of boundary shear stress distribution

Several methods are available for determining the intensity of shear stress on portions of the fixed boundary of a fluid flow. Direct methods include direct measurement of the force on a free-floating element of the wall or measurement of the movement of a spring-loaded element

of wall (33). Methods have been used which depend on measuring the rate of heat or mass transfer from a wall element (34,35). While each of these methods presents considerable difficulties in technique, it can be used in an empirical way so as not to depend on assumptions regarding the nature of the flow or of the velocity profile. Methods which involve assumptions regarding the nature of the velocity profile include the use of Preston or Stanton tubes, or simply of careful determination of the velocity profile near the wall (36).

No systematic program of boundary shear stress observation was included in the present study, although any extension of this work should certainly include such a program. In its absence, an estimate of shear stress distribution was attempted by using the velocity data given in Table 2, as follows.

The logarithmic velocity distribution law may be written as

$$u = m \log (y/y_0) \quad (14)$$

where

$$m = 2.303 u_* / \kappa = \frac{2.303}{\kappa} \sqrt{\frac{\tau_0}{\rho}}, \quad (15)$$

and κ is von Karman's constant.* Thus, if velocity profiles are observed in the vicinity of the bed or wall, and these profiles are closely enough spaced about the perimeter, the distribution of m can be determined. Integrating m^2 over the perimeter will then allow the mean value of κ to be determined, since $\overline{\tau_0}$ is known.

Thus the distribution of m^2 about the perimeter gives a measure

*It is well known that κ is not constant, depending in some unknown way on characteristics of the flow. It can therefore be thought of as defined locally along the boundary (except possibly in a corner) if τ_0 is similarly defined.

of τ_o/κ^2 and if κ^2 is assumed constant, m^2 gives a measure of shear stress distribution. Unfortunately, the data of Table 2 are not adequate to define the variation of m^2 along the perimeter, although m^2 did seem to vary rather widely and to show higher values near the center of the bed in the split-bed cases. How much of this is variation in local shear stress, how much is variation in κ , and how much is just experimental error, is not known. The method is indirect at best, and depends on having very carefully measured velocity profiles near the bed. For future work, a more direct method would be desirable, so that local variations in τ_o can be separated from local variations in κ .

V. DISCUSSION OF RESULTS: HYDRAULIC SUBDIVISION OF THE CROSS SECTION

There are several situations of flow over laterally varying roughness in which this variation is taken into account. In attempting to account for this variation it is customary to divide the cross sections of flow and to consider the resulting parts hydraulically independent. This is true in calculating the velocities and discharge of a river in flood, where the flow along a vegetated flood plain would usually be treated separately from that within the channel proper. Such a division is incorporated in the side-wall correction procedure widely used to reduce the results of observations in laboratory flumes to the values which would be obtained in the corresponding two-dimensional case. It is appropriate to review some of the hydraulic subdivision schemes which have been proposed, in particular of the side-wall correction procedures of Johnson (10), in order to determine the usefulness of such a process in predicting the resistance to flow in channels similar to that of the split bed runs of the present study.

1. Basic considerations

The argument supporting most methods of hydraulic subdivision goes as follows: steady, uniform flow can be partitioned in such a way that each zone of flow is associated with a segment of perimeter of constant roughness, and that each of these zones is hydraulically independent of the others. Each such zone may then be analyzed alone, and the customary equations applied. In the case of side-wall corrections, for example, the resulting values of f , r , etc. for the zone associated with the bed are supposedly equivalent to those for an infinitely wide channel of the same bed material.

In the past it has been tacitly assumed that the dividing surfaces

should be everywhere normal to curves of equal mean downstream velocity (i.e., they should be parallel to $\text{grad } \bar{u}$). Indeed, this assumption was stated by Leighly (37), who proceeded on the basis of it to find the distribution of the boundary shear stress and of the eddy diffusivity coefficient for several streams. Accordingly, the velocity surveys for runs 22, 23, and 36 were used to divide the flow and the pertinent hydraulic parameters were calculated.* The partition is indicated by dotted lines in Figures 11, 12, and 13, and the results of the calculations are given in Table 5.

In none of the three runs is there good agreement between the values calculated by the Leighly assumptions and those predicted by equation 5 for the smooth parts of the perimeter. In fact, the calculated friction factor for each wall agrees much more closely with that for the adjacent bed section than it does with the identical opposite side-wall. Thus it is obvious that the Leighly method of partitioning the flow is incorrect, in that the zones it produces are not independent. Hence, more careful consideration of the partition idea is in order.

Consider a turbulent flow which is statistically steady and uniform, and at any point within it choose local coordinates so that the x-axis is downstream, the y-axis is parallel to $\text{grad } \bar{u}$, and positive toward increasing velocities (i.e., in general not normal to the bed). The z-axis will then lie parallel to the isotachs (Fig. 15). As is customary, the x-, y-, and z-components of velocity are represented by u, v, and w, respectively. Each one (as well as the pressure, p) may be

*It should be noted that this differs from the usual side-wall correction procedure, for which detailed velocity traverses are rarely available. Instead, the wall roughness is assumed and the partitioning of the cross sectional area is computed. See Section 2 below.

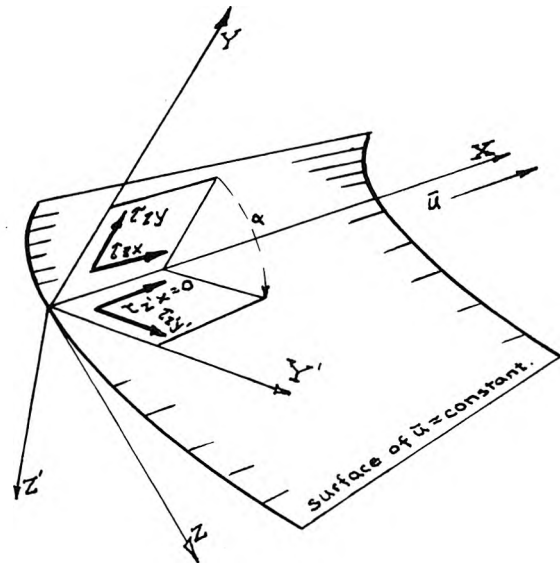


Figure 15. Local coordinate system.

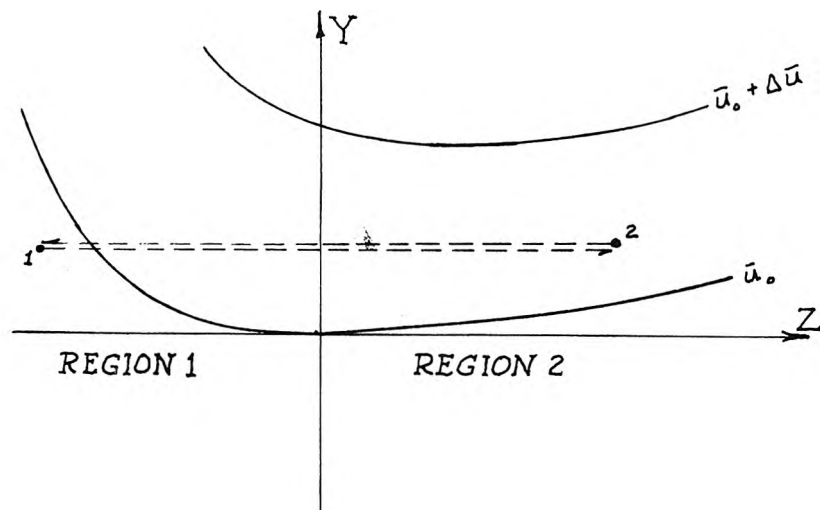


Figure 16. Momentum interchange with asymmetric isotachs.

Table 5

RESULTS OF PARTITIONING RUNS 22, 23, and 36 BY LEIGHTLY METHOD										
Section or Zone	Material	Estimated Roughness Height ϵ_i ft	Area of Associated Flow Zone A_i ft ²	Mean Velocity u_i fps	Hydraulic Radius r_i ft	Relative Roughness E_i	Reynolds Number R_i	Friction Factor f_i calc. by defin.	Friction Factor f_i predic. eq.(5) or (6)	$\frac{f_i \text{ calc.}}{f_i \text{ pred.}}$
Run 22: T = 23.3° C. S = 1.00% (Split bed)										
Left wall	Glass	10^{-6}	.0527	3.38	.1766	1.4×10^{-6}	238,000	.0400	.0137	2.92
Left 1/2 bed	Painted wood	10^{-4}	.0608	3.07	.1353	1.8×10^{-4}	167,000	.0371	.0146	2.54
Right 1/2 bed	Gravel	.0833	.0776	2.36	.1826	.1140	173,000	.0848	.1298*	0.65
Right wall	Glass	10^{-6}	.0681	2.61	.2337	1.1×10^{-6}	245,000	.0887	.0136	6.52
Entire channel	-	-	.2592	2.80	.1771	-	215,300	.0587	-	-
Run 23: T = 25.6° C. S = 2.52% (Split bed)										
Left wall	Glass	10^{-6}	.0125	4.08	.0833	3.0×10^{-6}	130,000	.0324	.0153	2.12
Left 1/2 bed	Painted wood	10^{-4}	.0417	4.14	.0942	2.7×10^{-4}	149,000	.0355	.0149	2.38
Right 1/2 bed	Gravel	.0833	.0595	2.59	.1372	.1518	135,000	.1324	.1567*	0.85
Right wall	Glass	10^{-6}	.0102	1.64	.0723	3.5×10^{-6}	49,700	.1740	.0187	9.31
Entire channel	-	-	.1239	3.18	.1063	-	157,700	.0685	-	-
Run 36: T = 22.7° C. S = 0.834% (Full rough bed)										
Left wall	Painted steel	10^{-4}	.0663	2.74	.1761	1.4×10^{-4}	189,800	.0503	.0143	3.52
Bottom	Gravel	.0833	.2062	2.25	.236	.0884	272,000	.1000	.1104*	0.91
Right wall	Painted steel	10^{-4}	.0575	2.90	.1525	1.6×10^{-4}	174,300	.0389	.0145	2.68
Entire channel	-	-	.3300	2.54	.202	-	192,300	.0672	-	-
Run 36: Partitioned by Johnson-Brooks side-wall correction procedure										
Left wall	Painted steel	10^{-4}	.025	2.64	.0664	3.8×10^{-4}	68,900	.0206	-	-
Bottom	Gravel	.0833	.280	2.64	.321	.0649	333,000	.0986	.0928	1.060
Right wall	Painted steel	10^{-4}	.025	2.64	.0664	3.8×10^{-4}	68,900	.0206	-	-
Entire channel	-	-	.330	2.64	.202	-	209,700	.0620	-	-

* Hydrodynamically rough, equation 6; all others are from equation 5.

NOTE: Discharge data for Leighly method are from velocity distributions and differ slightly from those in table 1 and those in Johnson-Brooks example.

divided into a mean portion (overscored) and a fluctuating portion (primed); e.g., $u = \bar{u} + u'$, and $\overline{u'} = 0$.

For the steady flow situation, the Reynolds equation for the x component becomes

$$\rho \left(\bar{u} \frac{\partial \bar{u}}{\partial x} + \bar{v} \frac{\partial \bar{u}}{\partial y} + \bar{w} \frac{\partial \bar{u}}{\partial z} \right) = -\rho g S - \frac{\partial \bar{p}}{\partial x} + \frac{\partial}{\partial x} \left[\mu \frac{\partial \bar{u}}{\partial x} - \rho \overline{u'^2} \right] + \frac{\partial}{\partial y} \left[\mu \frac{\partial \bar{u}}{\partial y} - \rho \overline{u'v'} \right] + \frac{\partial}{\partial z} \left[\mu \frac{\partial \bar{u}}{\partial z} - \rho \overline{u'w'} \right], \quad (15)$$

where the first term in each bracket on the right hand side is a true mean stress term, the last term (the so-called Reynolds stress) being a measure of momentum advected by turbulent velocity fluctuations. To speak of $\rho \overline{u'v'}$ as a stress is not strictly correct, but it is established usage and is satisfactory so long as the distinction between true and Reynolds stresses is kept in mind. I.e., we write

$$\tau_{ji} = \frac{\mu \partial u_i}{\partial x_j} - \rho \overline{u'_i u'_j}, \quad (16)$$

where τ_{ji} is the stress in the i direction on the plane normal to the j axis.

For uniform flow we have

$$\frac{\partial u_i}{\partial x} = 0 \quad \text{and} \quad \frac{\partial}{\partial x} (\overline{u'_i u'_j}) = 0$$

and because the z-axis parallels the isotachs, $\partial \bar{u} / \partial z = 0$. Using these relations, equation 15 becomes

$$\rho \left(\bar{v} \frac{\partial \bar{u}}{\partial y} \right) = -\rho g S - \frac{\partial \bar{p}}{\partial x} + \frac{\partial}{\partial y} \left[\mu \frac{\partial \bar{u}}{\partial y} - \rho \overline{u'v'} \right] + \frac{\partial}{\partial z} \left[-\rho \overline{u'w'} \right], \quad (17)$$

where now

$$\tau_{xx} = 0, \quad (18)$$

$$\tau_{yx} = \frac{\mu \partial \bar{u}}{\partial y} - \rho \overline{u'v'}, \quad (19)$$

$$\tau_{zx} = -\rho \overline{u'w'}. \quad (20)$$

Now if the z-plane is considered to divide the flow in the neighborhood of the origin into two parts, the resistance offered one part by the other is measured by τ_{zx} , which does not in general vanish under the assumptions merely of steady uniform flow. (If the flow is laminar, both τ_{zx} and τ_{zy} vanish, so that surfaces normal to the isotachs will be surfaces of zero shear, both longitudinal and transverse.)

If the dividing planar element is rotated through an angle α about the x-axis, and y'- and z'-axes are defined as shown in Figure 15, then the x-component of shear on the element, $\tau_{z'x}$, will vanish if $\tan \alpha = -\tau_{zx}/\tau_{yx}$, whereas in general the y'-component, $\tau_{z'y'}$, will not. Thus it cannot be assumed that both components vanish simultaneously.

We have thus shown that with each point within the flow there can be associated a planar element of such orientation that the longitudinal shear stress on it vanishes. In the absence of internal singularities in this set of oriented planar elements, it is therefore possible (conceptually, at least) to start at any point and construct a single continuous surface which is everywhere tangent to the planar elements, and which, therefore, will be a surface of zero longitudinal shear. Hence, if the starting points selected are points on the boundary at which the roughness changes (e.g., from the corners, and in the split-bed runs, from the point where the rocks meet the board), the flow will have been divided into zones between which there is no net interchange of momentum in the longitudinal direction. Such a division is always possible, but as shown above, these surfaces will not in general fall normal to the equal velocity surfaces.

To visualize a situation in which τ_{zx} on an element normal to the isotachs does not vanish, consider first a simple situation in which it does, namely that of two-dimensional flow. Here the isotachs are

straight, and the momentum interchange across any z-plane can be seen to be zero by symmetry. That is, the parcels of fluid carried across the plane in one direction have the same momentum, on the average, as those carried in the opposite direction. The same can be said in the case of a circular pipe, in which case the z-plane at any point will be a radial plane, and while individual parcels may now have different velocities $\bar{u} + u'$ from having been carried different distances to cross the z-plane, symmetry still prevails so that the net transport of momentum is seen to be zero.

If, however, the isotachs are not symmetrical with respect to the z-axis in the neighborhood of a particular point (Figs. 15 and 16), there will be a systematic bias in the distribution of u' -values for the parcels being carried across the z-plane. In Figure 16, parcels such as that labelled "2" will have higher velocities than their counterparts (labelled "1" in the figure), so there will be a net turbulent transport of momentum. In the figure, the flow in region 2 will experience a drag from the flow in region 1. The magnitude of this effect cannot be estimated, but if the mixing length concept has any physical reality, the fact that calculated values of the mixing length are of the order of tenths of feet rather than thousandths, say, means that even modest asymmetry of the isotachs may make the momentum flux significant.

If there is a secondary circulation such that $\bar{w} \neq 0$, the above argument is not affected. This can be seen by noticing that only the turbulent transport of momentum, $-\rho \overline{u'w'}$ is involved in the Reynolds stress, and observing that $\overline{u'w'} = \overline{u'w} - \overline{u'\bar{w}} = \overline{u'w}$, regardless of the value of \bar{w} .

Consideration of Figures 11 - 13 will show that asymmetry of

the isotachs could occur over much of the cross section, including the regions in which the zero-shear surfaces might be expected to fall. Thus it is quite reasonable that partition of the flow along lines normal to isovels should have produced such inconsistent results as Table 5 showed.

There still remain some physical objections to the partition concept, however, arising from the fact demonstrated above, that it is not necessarily true that because the longitudinal shear stress ($\tau_{z,x}$) has vanished, the transverse shear stress ($\tau_{z,y}$) must have vanished also. Thus, for example, the flow in a zone associated with a side-wall properly separated by a surface of zero longitudinal shear, is still influenced by the slower flow over the rough bottom, not directly by a longitudinal drag, but by the other component of shear on the dividing surface. The nature of this influence is not clear, but it seems likely that it might significantly modify the velocity distributions and turbulence properties of the zones, so as to make questionable the use of equations developed for quite a different flow situation (the Karman-Prandtl equations, for example). An additional point of concern is that these zones, particularly those associated with side-walls, may be more nearly triangular than rectangular, and as Keulegan (12) has pointed out, the shape of the flow section must be accounted for, although it would be quite difficult to decide on an appropriate value of his shape factor, β , for a narrow triangular section with only one wall!

2. Procedure

Thus far the general rationale of hydraulic subdivision of flow has been considered. However, the side-wall correction method as actually practiced departs somewhat from the rationale, so a detailed examination of the procedure is in order. For simplicity, consider steady, uniform

flow in a rectangular open channel whose bed is of one roughness and whose sides are both of another roughness. Of the physical quantities associated with the overall flow, those which are common to all zones are ρ , ν , g , and S , while those already known separately for the zones are P and ϵ . Therefore, the quantities to be partitioned may be taken as u , A , f , and r . Because of the symmetry about the vertical center line of the channel, the two wall sections will be considered together. A subscript w will apply to those quantities pertaining to the zones associated with the walls, subscript b to those pertaining to the zones associated with the bed, and no subscript to overall values.

Considering overall, wall-associated, and bed-associated quantities, there are twelve variables, four for each zone. Four of these, usually the overall quantities, are known. The following eight relations* can be written among the eight remaining variables

$$\text{Geometric continuity: } A_w + A_b = A \quad (21a)$$

$$\text{Flow continuity: } A_w u_w + A_b u_b = Au \quad (21b)$$

$$\text{Definition: } r_w = A_w / P_w \quad (21c)$$

$$\text{Definition: } r_b = A_b / P_b \quad (21d)$$

$$\text{Definition: } f_b = 8gr_b S / u_b^2 \quad (21e)$$

*Various authors differ in details of their procedure. Einstein (2) uses Manning's equation to define both the bed and bank roughness, and estimates the bank roughness n_w independently. Johnson (25) prefers the Karman-Prandtl resistance equation for the bed, because it is better able to account for the effects of temperature, and the roughness coefficient is dimensionless. The wall roughness, f_w , is usually estimated in his method, too; it need not be, however. Brooks (27, 38) has developed a modification which permits direct solutions for the situation in which the walls may be assumed to be hydrodynamically smooth, and he also considers (38, p. 236 ff) the effect of assuming some value for u_w/u other than unity.

Definition: $f_w = 8gr_w S/u_w^2$ (21f)

Empirical relation: $f_w = \phi_1(r_w, u_w; \nu, \epsilon_w)$ (21g)

Empirical relation: $f_b = \phi_2(r_b, u_b; \nu, \epsilon_b)$ (21h)

However, since the goal of most series of experiments is to investigate the functional relation which describes the behavior of f_b , it is begging the question to assume one. Hence, we are short one equation, and the assumption usually made to meet this deficiency is that $u_w = u_b$. While this is not strictly justifiable, Brooks (38) showed that assuming $u_w/u = 0.9$ gave results only a few percent different than did assuming $u = u_w$. The results of the velocity survey for run 36 were used to calculate the actual values of u_w/u for various arbitrarily chosen wall-associated zones. The zones measured are indicated in Figure 17, and the results are as follows:

Division Number	A_w/A	u_w/u
1	0.0245	0.868
2	0.0735	0.996
3	0.147	1.097
L4	0.196	1.244

Here, area L4 is a triangle chosen to approximate the left zone defined by the Leighly method.

Thus for small values of A_w/A , the assumption of $u_w = u$ is not a bad one. When A_w/A is large, as would be the case when wall and bed roughnesses are nearly the same, the similarity in roughness itself would tend to result in u_w/u not greatly different from unity.

It is interesting to note that nowhere in the side-wall correction procedure (as contrasted with the rationale) is the location of

the dividing surfaces specified, much less that they be required to be surfaces of zero shear. Indeed, if f_b and f_w are eliminated among equations 21 e, f, g, and h, and if equations 21 c and d are used to eliminate r_w and r_b , the resulting four equations in A_w , A_b , u_w , and u_b will be seen to reflect nothing of the internal nature of the flow, and the solution is seen to depend, for its recognition of physical reality, only on the particular resistance functions which are chosen. Since it is debatable whether these functions can realistically describe the behavior of flow in a channel with rigid boundaries but drastically non-regular shape, it is difficult to see that the procedure has any physical significance at all except in an approximate way. Thus the excellent results of its application to the data of the present study (see Chapter III, especially Figure 10), must for the time being be considered fortuitous, and use of the procedure in situations very different from those in which it is previously known to work should be undertaken with caution.

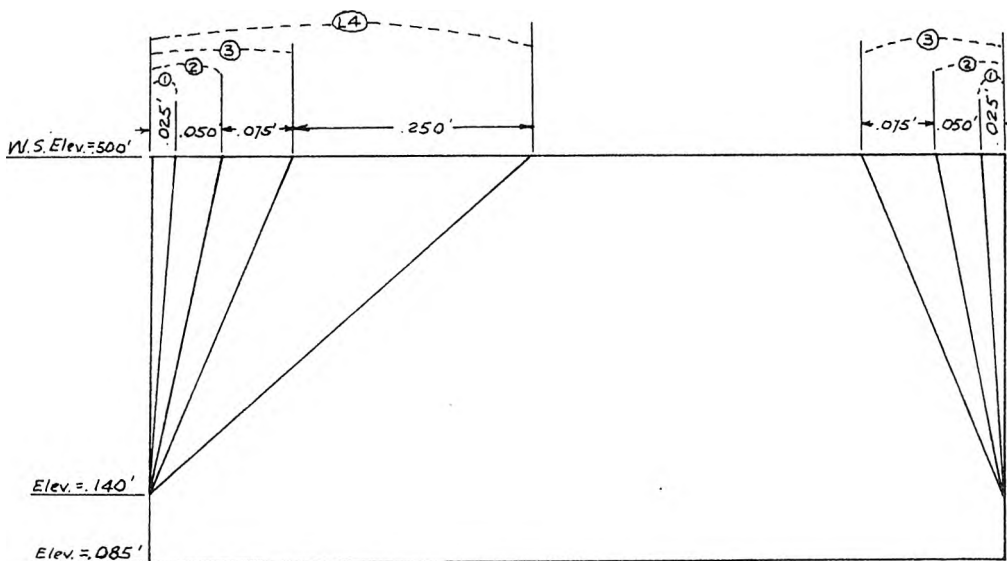


Figure 17. Arbitrarily chosen wall-associated zones, run 36.

VI. SUMMARY AND CONCLUSIONS; SUGGESTIONS FOR FUTURE WORK

1. Summary

Steady, uniform, open-channel flow was established at various depths and velocities over two types of beds, one rough over the entire width of a laboratory flume, the other rough only over half the width and smooth over the other half. Friction factors were determined for these flows, and detailed velocity distributions were measured in three runs. The friction factors were compared with those predicted by the Karman-Prandtl equations, and the velocity traverses were used to investigate the existence of secondary circulations and to assess the validity of methods of subdividing such a flow into hydrodynamically independent parts.

2. Conclusions

The following conclusions may be stated:

A. The bed friction factors determined for flow over the rough bed are consistent with the Karman-Prandtl equation for turbulent flow in rough, two-dimensional open channels, if the side-wall correction procedure of Johnson is applied, this consistency having been observed for relative roughnesses ranging from 0.09 to 0.18.

B. The overall friction factors found for flow over a bed rough on only one half varied consistently with relative roughness or hydraulic radius. They can be estimated within about six percent by weighting equally the values for the rough bed runs and for smooth, two-dimensional channels of the same hydraulic radius. They can be estimated only to within 30 percent of the true value by weighting the values for smooth and rough two-dimensional channels of the same hydraulic radius, if the latter values are taken from the Karman-Prandtl equations and if the

weighting is in proportion to the roughness distribution over the wetted perimeter. Values estimated by this latter method averaged about 10 percent high.

C. In considering methods of subdividing stream cross sections into hydrodynamically independent zones, it was concluded that surfaces of subdivision which lie normal to surfaces of equal mean downstream velocity are generally not surfaces of zero longitudinal shear stress (where the term shear stress is taken to include the advected turbulent momentum described as Reynolds stresses). Surfaces of zero longitudinal shear do exist; however, they do not necessarily divide the flow into independent parts because the transverse shear stress will not in general vanish on these surfaces. Therefore, subdivision of a turbulent flow into hydrodynamically independent zones is not in general possible, because the turbulence generated at the bed is undoubtedly diffused throughout the channel. Thus, while the customary side-wall correction procedure seems to be adequate for eliminating the effect of smooth side walls on the flow over a rough bed, this adequacy must be considered fortuitous, as there is no firm rational basis for the individual steps of the procedure.

D. The detailed velocity profiles for three runs indicate the existence of strong secondary circulation of the general pattern described by Prandtl as being "of the second kind".

3. Suggestions for future work

It was noted previously (p. 31) that the results of the present study could probably be extended to split beds of different roughness. The procedure mentioned was to determine the friction factor for a two-dimensional channel of the new roughness, to change it to one for a

channel of the required shape by using the side-wall correction procedure in reverse, and then to take the average of the friction factor thus modified and the friction factor for a smooth, two-dimensional channel of the same hydraulic radius. It remains to be seen, however, whether such a procedure would be effective for beds of other lateral distributions of roughness, and the assumption that it would be effective necessarily relies on an assumption of the validity of the side-wall correction procedure. Thus it is apparent that not only does considerably more work need to be done with different bed roughness sizes and distributions and with a wider range of width/depth ratios, but also a closer look must be taken at the internal nature of the flow. For example, more thorough and more precise velocity distribution measurements are needed, and the velocity measurements should be extended to include determinations of direction of flow, so as to shed more light on the secondary circulation patterns. These patterns are more than a curiosity, because it seems likely that they play some part in the interaction of different portions of the flow, even if not in the advection of turbulent momentum.

Of a more fundamental nature are observations (1) of the spatial variation of the turbulent velocity fluctuation correlations (or of the Reynolds stresses) or of the mixing length; (2) of the nature of the boundary layer in a wide channel in the neighborhood of a break in boundary roughness; (3) of the effect on a boundary layer of turbulent fluctuations arising from a nearby wall of different roughness (as in a closed wide channel with different roughnesses on opposite walls).

In addition to all the above, it should be remembered that the laterally varying roughness found in nature is interrelated with the transport of sediment. Once the questions mentioned above are extended

to flows transporting sediment, they become very much more complicated. This is particularly true of those questions relating to local properties of the flow.

Particular projects which should be undertaken next might include (1) determinations of velocity and Reynolds stress distributions in a wind tunnel approximating a two-dimensional channel, in which one or both walls were divided into respect to roughness or in which one wall were smooth and the other rough; (2) determination of the velocity vector distribution throughout an open-channel flow over laterally varying roughness; and (3) determination of the distribution of boundary shear stress about the perimeter.

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APPENDIX

Summary of Notation

In the following summary, the page number refers to the page on which each symbol is first used or defined.

Symbol	Description	Page
A	Area of cross section or of a zone thereof	51
A(y)	Area of surface which is actually water	21
b	Width of channel	13
b_g	Width of rough part of channel	13
d	Depth of flow	14
E	Relative roughness	4
f	Darcy-Weisbach friction factor	4
f_A	Friction factor estimated by Method A	31
f_B	Friction factor estimated by Method B	31
f_r	Friction factor for flow in rough two-dimensional channels	4
f_R	Observed friction factor for rough bed	26
f_{ro}	Friction factor for flow in rough circular sections	4
f_s	Friction factor for flow in smooth two-dimensional channels	4
f_S	Observed friction factor for split bed	26
f_{so}	Friction factor for flow in smooth circular sections	4
F	Froude No.	14
g	Acceleration of gravity	4
m	Slope of semilogarithmic velocity distribution	42
P	Perimeter	51
p	Pressure	47
Q	Fluid discharge	14

r	Hydraulic radius	4
r_o	Radius of spherical or hemispherical bed roughness element	20
R	Reynolds number	5
S	Energy gradient	4
u	Mean downstream velocity	4
u_*	Shear velocity	5
v	Velocity in y direction	45
w	Velocity in z direction	45
W	Ratio of width, b, to depth, d	4
x	Downstream direction	45
y	Direction normal to isotachs	45
y_o	Arbitrary length constant in equation 14	42
z	Direction tangent to isotachs	45
α	Angle through which dividing planar element is rotated	48
β	Shape factor for polygonal channels (Keulegan, ref. 12)	6
ϵ	Height of boundary roughness elements	4
θ	Roughness distribution ratio	14
κ	von Karman's constant	42
μ	Dynamic viscosity	47
ν	Kinematic viscosity	5
ρ	Density of fluid	13
τ_{ji}	Shear stress in the i direction on the plane normal to the j axis	47
τ_o	Boundary shear stress	42
φ	Functional relationship defined in equation 10	15

Subscripts and other supplementary symbols systematically used

b Quantities pertaining to zones associated with the bed (A, P, f, u)

w Quantities pertaining to zones associated with the walls (A, P, f, u)

— Mean value (of velocity, u, v, or w; or of pressure, p)

' Fluctuating value (of velocity, u, v, or w)

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